CHAPTER 7

COORDINATE GEOMETRY

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. The point P on x-axis equidistant from the points $A(-1,0)$ and $B(5,0)$ is

(a) (2, 0) (b) (0, 2)

(c) (3, 0) (d) $(-3,5)$

Ans : [Board 2020 OD Standard]

Let the position of the point P on x-axis be $(x,0)$, then

$$PA^2 = PB^2$$

$$(x+1)^2+(0)^2 = (5-x)^2+(\ )^2$$

$$x^2+2x+1 = 25+x^2-10x$$

$$2x+10x = 25-1$$

$$12x = 24 \Rightarrow x = 2$$

Hence, the point $P(x,0)$ is (2, 0).

Thus (a) is correct option.

Alternative :

You may easily observe that both point $A(-1,0)$ and $B(5,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-1,0)$ and $B(5,0)$.

$$x = \frac{-1+5}{2} = 2$$

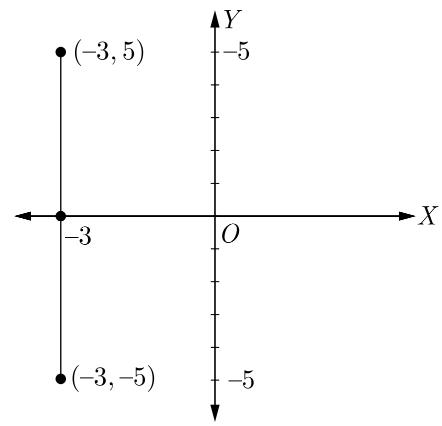
2. The co-ordinates of the point which is reflection of point $(-3,5)$ in x-axis are

(a) (3, 5) (b) (3, -5)

(c) $(-3,-5)$ (d) $(-3,5)$

Ans : [Board 2020 OD Standard]

The reflection of point $(-3,5)$ in x- axis is $(-3,-5)$.



Thus (c) is correct option.

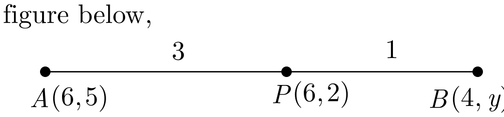
3. If the point P (6, 2) divides the line segment joining $A(6,5)$ and $B(4,y)$ in the ratio $3:1$ then the value of y is

(a) 4 (b) 3

(c) 2 (d) 1

Ans : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below,



Here, $x_1 = 6,\ y_1 = 5$

and $x_2 = 4\ \ y_2 = y$

Now $y = \dfrac{my_2+ny_1}{m+n}$

$$2 = \frac{3\times y+1\times 5}{3+1}$$

$$2 = \frac{3y+5}{4}$$

$$3y+5 = 8$$

$$3y = 8-5 = 3 \Rightarrow y = 1$$

Thus (d) is correct option.

4. The distance between the points $(a\cos\theta + b\sin\theta,\ 0)$, and $(0,\ a\sin\theta - b\cos\theta)$ is

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

Ans : [Board 2020 Delhi Standard]

We have $x_1 = a\cos\theta + b\sin\theta$ and $y_1 = 0$

and $x_2 = 0$ and $y_2 = a\sin\theta - b\cos$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2$$
$$= (-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$
$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + $$
$$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$
$$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$
$$= a^2 \times 1 + b^2 \times 1 = a^2 + b^2$$

Thus $d^2 = a^2 + b^2$

$$d = \sqrt{a^2 + b^2}$$

Therefore (c) is correct option.

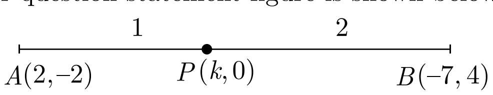
5. If the point $P(k,\ 0)$ divides the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ in the ratio 1 : 2, then the value of k is

(a) 1 (b) 2

(c) -2 (d) -1

Ans : [Board 2020 Delhi Standard]

As per question statement figure is shown below.



$$k = \frac{1(-7) + 2(2)}{1 + 2} \qquad \left(x = \frac{mx_2 + nx_1}{m + n}\right)$$
$$= \frac{-7 + 4}{3} = \frac{-3}{3} = -1$$

Thus $k = -1$

Thus (d) is correct option.

6. The coordinates of a point A on y-axis, at a distance of 4 units from x-axis and below it are

(a) (4, 0) (b) (0, 4)

(c) $(-4, 0)$ (d) $(0,\ -4)$

Ans :
[Board 2020 Delhi Basic]

Because the point is 4 units down the x-axis i.e., co-ordinate is -4 and on y-axis abscissa is 0. So, the coordinates of point A is $(0, -4)$.

Thus (d) is correct option.

7. The distance of the point $(-12,\ 5)$ from the origin is

(a) 12 (b) 5

(c) 13 (d) 169

Ans :

The distance between the origin and the point (x, y) is $\sqrt{x^2 + y^2}$.

Therefore, the distance between the origin and point $(-12, 5)$

$$d = \sqrt{(-12 - 0)^2 + (5 - 0)^2}$$
$$= \sqrt{144 + 25} = \sqrt{169}$$
$$= 13 \text{ units}$$

Thus (c) is correct option.

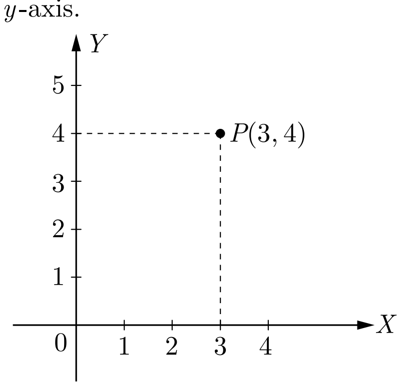
8. Distance of point $P(3, 4)$ from x-axis is

(a) 3 units (b) 4 units

(c) 5 units (d) 1 units

Ans : [Board 2020 Delhi Basic]

Point $P(3, 4)$ is 4 units from the x-axis and 3 units from the y-axis.



Thus (b) is correct option.

9. The distance of the point $P(-3, -4)$ from the x-axis (in units) is

(a) 3 (b) -3

(c) 4 (d) 5

Ans :
[Board 2020 SQP Standard]

Point $P(-3, -4)$ is 4 units from the x-axis and 3 units from the y-axis.

Thus (c) is correct option.

10. If $A(\frac{m}{3}, 5)$ is the mid-point of the line segment joining the points $Q(-6,7)$ and $R(-2,3)$, then the value of m is

(a) -12 (b) -4

(c) 12 (d) -6

Ans : [Board 2020 SQP Standard]

Given points are $Q(-6,7)$ and $R(-2,3)$

Mid point $A(\frac{m}{3}, 5) = \left(\frac{-6-2}{2}, \frac{7+3}{2}\right)$

$= (-4, 5)$

Equating, $\frac{m}{3} = -4 \Rightarrow m = -12$

Thus (a) is correct option.

11. The mid-point of the line-segment AB is $P(0,4)$, if the coordinates of B are $(-2, 3)$ then the co-ordinates of A are

(a) (2, 5) (b) $(-2, -5)$

(c) (2, 9) (d) $(-2, 11)$

Ans : [Board 2020 OD Basic]

Let point A be (x, y).

Now using mid-point formula,

$$(0,4) = \left(\frac{x-2}{2}, \frac{y+3}{2}\right)$$

Thus $0 = \frac{x-2}{2} \Rightarrow x = 2$

and $4 = \frac{y+3}{2} \Rightarrow y = 5$

Hence point A is (2, 5).

Thus (a) is correct option.

12. x-axis divides the line segment joining $A(2,-3)$ and $B(5,6)$ in the ratio

(a) 2 : 3 (b) 3 : 5

(c) 1 : 2 (d) 2 : 1

Ans : [Board 2020 OD Basic]

Let point $P(x, 0)$ on x-axis divide the segment joining points $A(2, -3)$ and $B(5, 6)$ in ratio $k : 1$, then

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$0 = \frac{6k-3}{k+1}$$

$$6k = 3 \Rightarrow k = \frac{1}{2}$$

Therefore ratio is 1 : 2.

Thus (c) is correct option.

13. The point which divides the line segment joining the points $(8,-9)$ and $(2, 3)$ in the ratio $1 : 2$ internally lies in the

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

Ans : [Board 2020 SQP Standard]

We have $x_1 = 8$, $y_1 = -9$, $x_2 = 2$ and $y_2 = 3$.

and $m_1 : m_2 = 1 : 2$

Let the required point be $P(x, y)$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1 \times 2 + 2 \times 8}{1+2} = 6$$

and $$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 3 + 2(-9)}{1+2} = -5$$

Thus $(x, y) = (6, -5)$ and this point lies in IV quadrant.

Thus (d) is correct option.

14. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and $(2, y)$, then the value of y is

(a) 3 (b) -3

(c) 7 (d) 4

Ans : [Board 2020 Delhi Basic]

Since, centre is the mid-point of end points of the diameter.

$$(3, 5) = \left(\frac{4+2}{2}, \frac{7+y}{2}\right)$$

Comparing both the sides, we get

$$5 = \frac{7+y}{2}$$

$$7 + y = 10 \Rightarrow y = 3$$

Thus (a) is correct option.

15. If the distance between the points $A(4, p)$ and $B(1,0)$ is 5 units then the value(s) of p is(are)

(a) 4 only (b) -4 only

(c) ± 4 (d) 0

Ans : [Board 2020 Delhi Basic]

Given, points are $A(4, p)$ and $B(1,0)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$5 = \sqrt{(1-4)^2 + (0-p}$$
$$25 = 9 + p^2$$
$$p^2 = 25 - 9 = 16$$
$$p = \pm 4$$

Thus (c) is correct option.

16. If the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, then $\frac{1}{a} + \frac{1}{b}$ equals

(a) 1 (b) 2

(c) 0 (d) -1

Ans :

Let the given points are $A(a, 0)$, $B(0, b)$ and $C(1, 1)$. Since, A, B, C are collinear.

Hence, $\text{ar}(\Delta ABC) = 0$

$$\frac{1}{2}[a(b-1) + 0(1-0) + 1(0-b)] = 0$$
$$ab - a - b = 0$$
$$a + b = ab$$
$$\frac{a+b}{ab} = 1$$
$$\frac{1}{a} + \frac{1}{b} = 1$$

Thus (a) is correct option.

17. If the points $A(4, 3)$ and $B(x, 5)$ are on the circle with centre $O(2, 3)$, then the value of x is

(a) 0 (b) 1

(c) 2 (d) 3

Ans :

Since, A and B lie on the circle having centre O.

$$OA = OB$$
$$\sqrt{(4-2)^2 + (3-3)^2} = \sqrt{(x-2)^2 + (5-3)^2}$$
$$2 = \sqrt{(x-2)^2 + 4}$$
$$4 = (x-2)^2 + 4$$
$$(x-2)^2 = 0 \Rightarrow x = 2$$

Thus (c) is correct option.

18. The ratio in which the point $(2, y)$ divides the join of $(-4, 3)$ and $(6, 3)$, hence the value of y is

(a) $2:3, y = 3$ (b) $3:2, y = 4$

(c) $3:2, y = 3$ (d) $3:2, y = 2$

Ans :

Let the required ratio be $k:1$

Then, $$2 = \frac{6k - 4(1)}{k+1}$$

or $$k = \frac{3}{2}$$

The required ratio is $\frac{3}{2}:1$ or $3:2$

Also, $$y = \frac{3(3) + 2(3)}{3+2} = 3$$

Thus (c) is correct option.

19. The point on the x-axis which is equidistant from the points $A(-2, 3)$ and $B(5, 4)$ is

(a) (0, 2) (b) (2, 0)

(c) (3, 0) (d) $(-2, 0)$

Ans :

Let $P(x, 0)$ be a point on x-axis such that,

$$AP = BP$$
$$AP^2 = BP^2$$
$$(x+2)^2 + (0-3)^2 = (x-5)^2 + (0+4)^2$$
$$x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$$
$$14x = 28$$
$$x = 2$$

Hence required point is (2, 0).

Thus (b) is correct option.

20. C is the mid-point of PQ, if P is $(4, x)$, C is $(y, -1)$ and Q is $(-2, 4)$, then x and y respectively are

(a) -6 and 1 (b) -6 and 2

(c) 6 and -1 (d) 6 and -2

Ans :

Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$.

We have, $$\frac{4-2}{2} = y \Rightarrow y = 1$$

and $$\frac{4+x}{2} = -1 \Rightarrow x = -6$$

Thus (a) is correct option.

21. If three points $(0, 0)$, $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then λ equals

(a) 2 (b) -3

(c) -4 (d) None of these

Ans :

Let the given points are $A(0,0)$, $B(3,\sqrt{3})$ and $C(3,\lambda)$.

Since, ΔABC is an equilateral triangle, therefore

$$AB = AC$$

$$\sqrt{(3-0)^2+(\sqrt{3}-0)^2} = \sqrt{(3-0)^2+(\lambda-0)^2}$$

$$9+3 = 9+\lambda^2$$

$$\lambda^2 = 3 \Rightarrow \lambda = \pm\sqrt{3}$$

Thus (d) is correct option.

22. If $x-2y+k=0$ is a median of the triangle whose vertices are at points $A(-1,3)$, $B(0,4)$ and $C(-5,2)$, then the value of k is

(a) 2 (b) 4

(c) 6 (d) 8

Ans :

Coordinate of the centroid G of ΔABC

$$=\left(\frac{-1+0-5}{2}, \frac{3+4+2}{3}\right)$$

$$=(-2,3)$$

Since, G lies on the median, $x-2y+k=0$, it must satisfy the equation,

$$-2-6+k=0 \Rightarrow k=8$$

Thus (d) is correct option.

23. The centroid of the triangle whose vertices are $(3,-7)$, $(-8,6)$ and $(5, 10)$ is

(a) (0, 9) (b) (0, 3)

(c) (1, 3) (d) (3, 5)

Ans :

Centroid is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

i.e. $\left(\frac{3+(-8)+5}{3}, \frac{-7+6+10}{3}\right) = \left(\frac{0}{3}, \frac{9}{3}\right)$

$$=(0,3)$$

Thus (b) is correct option.

24. The distance of the point $P(2,3)$ from the x-axis is

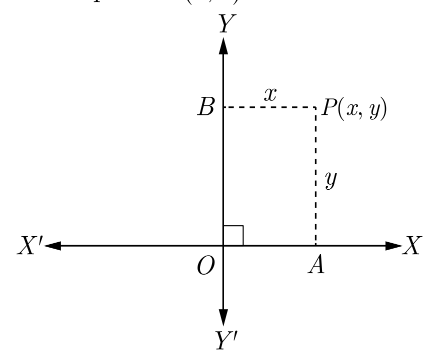
(a) 2 (b) 3

(c) 1 (d) 5

Ans :

We know that, if (x,y) is any point on the cartesian plane in first quadrant, then x is perpendicular distance from y-axis and y is perpendicular distance from x-axis.

Distance of the point $P(2,3)$ from the x-axis is 3.



Thus (b) is correct option.

25. The distance between the points $A(0,6)$ and $B(0,-2)$ is

(a) 6 (b) 8

(c) 4 (d) 2

Ans :

Distance between the points (x_1,y_1) and (x_2,y_2) is given as,

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Here, $x_1 = 0$, $y_1 = 6$ and $x_2 = 0$, $y_2 = -2$

Distance between $A(0,6)$ and $B(0,-2)$

$$AB = \sqrt{(0-0)^2+(-2-6}$$

$$= \sqrt{0+(-8)^2} = \sqrt{8^2} = 8$$

Thus (b) is correct option.

26. The distance of the point $P(-6,8)$ from the origin is

(a) 8 (b) $2\sqrt{7}$

(c) 10 (d) 6

Ans :

Distance between the points (x,y) and origin is given as,

$$d = \sqrt{x^2+y^2}$$

Distance between $P(-6,8)$ and origin is,

$$PO = \sqrt{(6)^2+(-8)^2} = \sqrt{36+64}$$

$$= \sqrt{100} = 10$$

Thus (c) is correct option.

27. The distance between the points (0, 5) and $(-5, 0)$ is

(a) 5 (b) $5\sqrt{2}$

(c) $2\sqrt{5}$ (d) 10

Ans :

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0,\ y_1 = 5$ and $x_2 = -5,\ y_2 = 0$

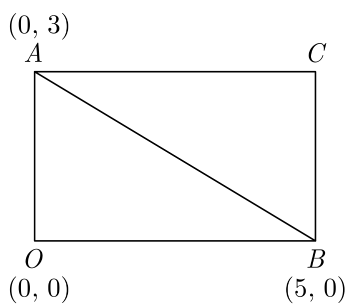
Distance between the points (0, 5) and $(-5, 0)$

$$d = \sqrt{[-5-0]^2 + [0-(-5)]^2}$$

$$= \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

Thus (b) is correct option.

28. If $AOBC$ is a rectangle whose three vertices are $A(0, 3)$, $O(0, 0)$ and $B(5, 0)$, then the length of its diagonal is



(a) 5 (b) 3

(c) $\sqrt{34}$ (d) 4

Ans :

Length of the diagonal is AB which is the distance between the points $A(0, 3)$ and $B(5, 0)$.

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0,\ y_1 = 3$, and $x_2 = 5,\ y_2 = 0$

Distance between the points $A(0, 3)$ and $B(5, 0)$

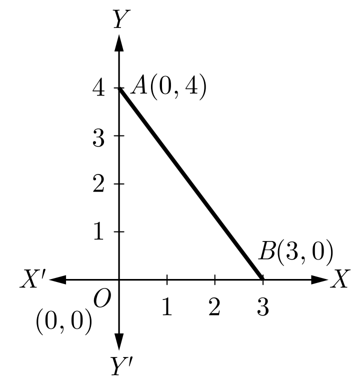
$$AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{25 + 9} = \sqrt{34}$$

Hence, the required length of its diagonal is $\sqrt{34}$.

Thus (c) is correct option.

29. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is



(a) 5 (b) 12

(c) 11 (d) $7 + \sqrt{5}$

Ans :

We have $OA = 4$

$OB = 3$

and $AB = \sqrt{3^2 + 4^2} = 5$

Now, perimeter of ΔAOB is the sum of the length of all its sides.

$$p = OA + OB + AB = 4 + 3 + 5 = 12$$

Hence, the required perimeter of triangle is 12. However you can calculate perimeter direct from diagram.

Thus (b) is correct option.

30. The point which lies on the perpendicular bisector of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$ is

(a) (0, 0) (b) (0, 2)

(c) (2, 0) (d) $(-2, 0)$

Ans :

We know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

Mid-point of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$

$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

Thus (a) is correct option.

31. If the point $P(2, 1)$ lies on the line segment joining points $A(4,2)$ and $B(8,4)$, then

(a) $AP = \frac{1}{3}AB$ (b) $AP = PB$

(c) $PB = \frac{1}{3}AB$ (d) $AP = \frac{1}{2}AB$

Ans :

Let, $AP:AB = m:n$

Using section formula, we have,

$$4 = \frac{8m+2n}{m+n}$$

and $$2 = \frac{4m+n}{m+n}$$

Solving these as linear equation, we get,

$$m = 1 \text{ and } n = 2$$

$$\frac{AP}{AB} = \frac{1}{2}$$

$$AP = \frac{1}{2}AB$$

Thus (d) is correct option.

32. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6,5)$ and $R(-2,3)$, then the value of a is

(a) -4 (b) -12

(c) 12 (d) -6

Ans :

Since $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the points $Q(-6,5)$ and $R(-2,3)$,

$$\left(\frac{a}{3}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$

$$\left(\frac{a}{3}, 4\right) = (-4, 4)$$

Now $$\frac{a}{3} = -4 \Rightarrow a = -12$$

Thus (b) is correct option.

33. The perpendicular bisector of the line segment joining the points $A(1,5)$ and $B(4,6)$ cuts the y-axis at

(a) (0, 13) (b) (0, – 13)

(c) (0, 12) (d) (13, 0)

Ans :

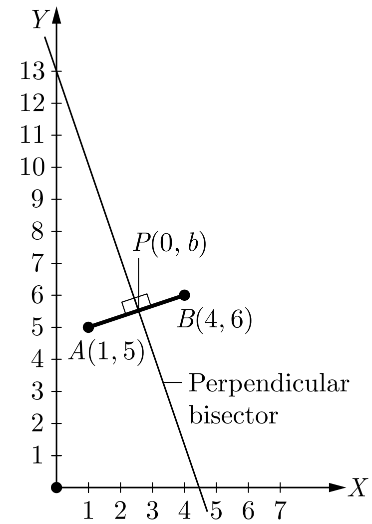
Let $P(0,b)$ be the required point. Since, any point on perpendicular bisector is equidistant from the end point of line segment.

i.e., $PA = PB$

$$\sqrt{(0-1)^2+(b-5)^2} = \sqrt{(0-4)^2+(b-6}$$

$$1 + b^2 - 10b + 25 = 16 + b^2 - 12b + 36$$

$$2b = 26 \Rightarrow b = 13$$



Thus (a) is correct option.

34. If the distance between the points $(4, p)$ and $(1, 0)$ is 5, then the value of p is

(a) 4 only (b) ± 4

(c) -4 only (d) 0

Ans :

According to the question, the distance between the points $(4, p)$ and $(1, 0)$ is 5.

i.e., $$\sqrt{(1-4)^2+(0-p)^2} = 5$$

$$\sqrt{(-3)^2+p^2} = 5$$

$$\sqrt{9+p^2} = 5$$

Squaring both the sides, we get,

$$9 + p^2 = 25$$

$$p^2 = 16 \Rightarrow p = \pm 4$$

Hence, the required value of p is ± 4.

Thus (b) is correct option.

35. Assertion : The value of y is 6, for which the distance between the points $P(2,-3)$ and $Q(10,y)$ is 10.

Reason : Distance between two given points $A(x_1, y_1)$ *and* $B(x_2, y_2)$ is given,

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$PQ = 10$$
$$PQ^2 = 100$$
$$(10-2)^2+(y+3)^2 = 100$$
$$(y+3)^2 = 100-64 = 36$$
$$y+3 = \pm 6$$
$$y = -3 \pm 6$$
$$y = 3, -9$$

Assertion (A) is false but reason (R) is true.

Thus (s) is correct option.

Fill in the Blank Questions

36. All the points equidistant from two given points A and B lie on the .... of the line segment AB.

Ans :

perpendicular bisector

37. The distance of a point from the y-axis is called its ....

Ans :

abscissa

38. The distance of a point from the x-axis is called its ....

Ans :

ordinate

39. The value of the expression $\sqrt{x^2+y^2}$ is the distance of the point $P(x,y)$from the ....

Ans :

origin

40. The distance of the point (p,q) from (a,b) is ....

Ans :

$\sqrt{(a-p)^2+(b-q)^2}$

41. If the area of the triangle formed by the vertices $A(x_1,y_1)\,B(x_2,y_2)$ and $C(x_3,y_3)$is zero, then the points A, B and C are ....

Ans :

collinear

42. A point of the form $(b,0)$ lies on ....

Ans :

x-axis

43. The distance of the point (x_1,y_1) from the origin is ....

Ans :

$\sqrt{x_1^2+y_1^2}$

44. A point of the form $(0,a)$ lies on ....

Ans :

y-axis

45. If the point $C(k,\ 4)$ divides the line segment joining two points $A(2,\ 6)$ and $B(5,1)$ in ratio 2 : 3, the value of k is ....

Ans : [Board 2020 Delhi Basic]

We have $m:n = 2:3$

By section formula,

$$\frac{mx_2+nx_1}{m+n} = x$$

Now, $$\frac{2\times5+3\times2}{2+3} = k \Rightarrow k = \frac{16}{5}$$

46. If points $A(-3,12)$, $B(7,6)$ and $C(x,9)$ are collinear, then the value of x is ....

Ans : [Board 2020 Delhi Basic]

If points are collinear, then area of triangle must be zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$
$$\frac{1}{2}[-3(6-9)+7(9-12)+x(12-6)] = 0$$
$$\frac{1}{2}(9-21+6x = 0$$
$$\frac{1}{2}(-12+6x) = 0$$
$$6x = 12 \Rightarrow x = 2$$

47. The co-ordinate of the point dividing the line segment joining the points $A(1,3)$ and $B(4,6)$ in the ratio 2 : 1 is ....

Ans : [Board 2020 OD Basic]

Let point $P(x,\ y)$ divides the line segment joining points $A(1,3)$ and $B(4,6)$ in the ratio 2 : 1.

Using section formula we have

$$(x,\ y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\ \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$(x,\ y) = \left(\frac{2\times4+1\times1}{2+1},\ \frac{2\times6+1\times3}{2+1}\right)$$

$$= \left(\frac{8+1}{3},\ \frac{12+3}{3}\right) = \left(\frac{9}{3},\ \frac{15}{3}\right) = (3,5)$$

Very Short Answer Questions

48. Find the distance of a point $P(x,y)$ from the origin.

Ans : [Board 2018]

Distance between origin (0, 0) and point $P(x,y)$ is

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(x-0)^2+(y-0)^2}$$
$$= \sqrt{x^2+y^2}$$

Distance between P and origin is $\sqrt{x^2+y^2}$.

49. If the mid-point of the line segment joining the points $A(3,4)$ and $B(k,6)$ is $P(x,y)$ and $x+y-10=0$, find the value of k.

Ans : [Board 2020 OD Standard]

If $P(x,y)$ is mid point of $A(3,4)$ and $B(k,6)$, then we have

$$\frac{3+k}{2} = x \text{ and } y = \frac{4+6}{2} = \frac{10}{2} = 5$$

Substituting above value in $x+y-10=0$ we have

$$\frac{3+k}{2}+5-10 = 0$$

$$\frac{3+k}{2} = 5$$

$$3+k = 10 \Rightarrow k = 10-3 = 7$$

50. Write the coordinates of a point P on x-axis which is equidistant from the points $A(-2,0)$ and $B(6,0)$.

Ans : [Board 2019 OD]

Since it is equidistant from the points $A(-2,0)$ and $B(6,0)$ then

$$AP = BP$$

$$AP^2 = BP^2$$

Using distance formula we have

$$[(x-(-2)]^2+(0-0)^2 = (x+6)^2+(0-0)^2$$
$$(x+2)^2 = (x+6)^2$$
$$x^2+4x+4 = x^2+12x+36$$
$$8x = -32$$
$$x = -4$$

Hence, required point P is $(-4,0)$.

Alternative :

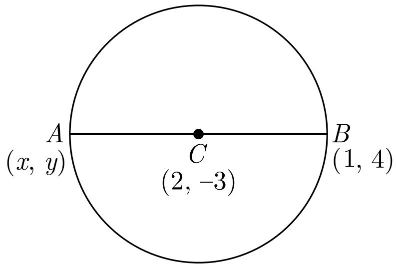
You may easily observe that both point $A(-2,0)$ and $B(6,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-2,0)$ and $B(6,0)$.

$$x = \frac{-2+6}{2} = 2$$

51. Find the coordinates of a point A, where AB is diameter of a circle whose centre is $(2,-3)$ and B is the point $(1,4)$.

Ans : [Board 2019 Delhi]

As per question we have shown the figure below. Since, AB is the diameter, centre C must be the mid point of the diameter of AB.



Let the co-ordinates of point A be (x,y).

x-coordinate of C,

$$\frac{x+1}{2} = 2$$
$$x+1 = 4 \Rightarrow x = 3$$

and y-coordinate of C,

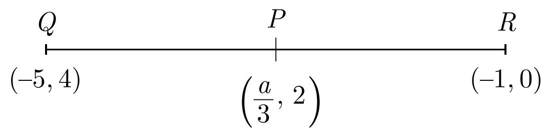
$$\frac{y+4}{2} = -3$$
$$y+4 = -6 \Rightarrow y = -10$$

Hence, coordinates of point A are $(3,-10)$.

52. Find the value of a, for which point $P(\frac{a}{3},2)$ is the midpoint of the line segment joining the Points $Q(-5,4)$ and $R(-1,0)$.

Ans : [Board Term-2 SQP 2016]

As per question, line diagram is shown below.



Since P is mid-point of QR, we have

$$\frac{a}{3} = \frac{-5+(-1)}{2} = \frac{-6}{2} = -3$$

Thus $a = -9$

53. The ordinate of a point A on y-axis is 5 and B has co-ordinates $(-3,1)$. Find the length of AB.

Ans : [Board Term-2 2014]

We have $A(0,5)$ and $B(-3,1)$.

Distance between A and B,

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1^{\ 2}}$$
$$= \sqrt{(-3-0)^2+(1-5)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25} = 5$$

54. Find the perpendicular distance of $A(5,12)$ from the y-axis.

Ans : [Board Term-2 2011]

Perpendicular from point $A(5,12)$ on y-axis touch it at $(0,12)$.

Distance between $(5,12)$ and $(0,12)$ is,

$$d = \sqrt{(0-5)^2+(12-12}$$
$$= \sqrt{25}$$
$$= 5 \text{ units.}$$

55. If the centre and radius of circle is $(3,4)$ and 7 units respectively,, then what it the position of the point $A(5,8)$ with respect to circle?

Ans : [Board Term-2 2013]

Distance of the point, from the centre,

$$d = \sqrt{(5-3)^2+(8-4)^2}$$
$$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Since $2\sqrt{5}$ is less than 7, the point lies inside the circle.

56. Find the perimeter of a triangle with vertices $(0,4)$, $(0,0)$ and $(3,0)$.

Ans : [Board Term-2, 2011]

We have $A(0,4), B(0,0)$, and $C(3,0)$.

$$AB = \sqrt{(0-2)^2+(0-4)^2} = \sqrt{16} = 4$$
$$BC = \sqrt{(3-0)^2+(0-0)^2} = \sqrt{9} = 3$$
$$CA = \sqrt{(0-3)^2+(4-0)^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5$$

Thus perimeter of triangle is $4+3+5 = 12$

57. Locate a point Q on line segment AB such that $BQ = \frac{5}{7}\times AB$. What is the ratio of line segment in which AB is divided?

Ans : [Board Term-2 2013]

We have $BQ = \frac{5}{7}AB$

$$\frac{BQ}{AB} = \frac{5}{7} \Rightarrow \frac{AB}{BQ} = \frac{7}{5}$$
$$\frac{AB-BQ}{BQ} = \frac{7-5}{5}$$
$$\frac{AQ}{BQ} = \frac{2}{5}$$

Thus $AQ:BQ = 2:5$

58. Find the distance of the point $(-4,-7)$ from the y-axis.

Ans : [Board Term-2 2013]

Perpendicular from point $A(-4,-7)$ on y-axis touch it at $(0,-7)$.

Distance between $(-4,-7)$ and $(0,-7)$ is

$$d = \sqrt{(0+4)^2+(-7+7)^2}$$
$$= \sqrt{4^2+0} = \sqrt{16} = 4 \text{ units}$$

59. If the distance between the points $(4,k)$ and $(1,0)$ is 5, then what can be the possible values of k.

Ans : [Board Term-2 2017]

Using distance formula we have

$$\sqrt{(4-1)^2+(k-0)^2} = 5$$
$$3^2+k^2 = 25$$
$$k^2 = 25-9 = 16$$
$$k = \pm 4$$

60. Find the coordinates of the point on y-axis which is nearest to the point $(-2,5)$.

Ans : [Board Term-2 SQP 2017]

Point $(0,5)$ on y-axis is nearest to the point $(-2,5)$.

61. In what ratio does the x-axis divide the line segment joining the points $(-4,-6)$ and $(-1,7)$? Find the coordinates of the point of division.

Ans : [Board Term-2 SQP 2017]

Let x-axis divides the line-segment joining $(-4,-6)$ and $(-1,7)$ at the point $P(x,y)$ in the ratio $1:k$.

Now, the coordinates of point of division P,

$$(x,y) = \frac{1(-1)+k(-4)}{k+1}, \frac{1(7)+k(-6)}{k+1}$$

$$= \frac{-1-4k}{k+1}, \frac{7-6k}{k+1}$$

Since P lies on x axis, therefore $y=0$, which gives

$$\frac{7-6k}{k+1} = 0$$

$$7-6k = 0$$

$$k = \frac{7}{6}$$

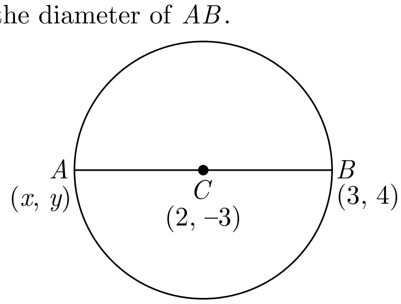
Hence, the ratio is $1:\frac{7}{6}$ or, $6:7$ and the coordinates of P are $\left(-\frac{34}{13}, 0\right)$.

TWO MARKS QUESTIONS

62. Find the coordinates of a point A, where AB is diameter of the circle whose centre is $(2,-3)$ and B is the point $(3,4)$.

Ans : [Board 2019 Delhi]

As per question we have shown the figure below. Since, AB is the diameter, centre C must be the mid point of the diameter of AB.



Let the co-ordinates of point A be (x,y).

x-coordinate of C,

$$\frac{x+3}{2} = 2$$

$$x+3 = 4 \Rightarrow x = 1$$

and y-coordinate of C,

$$\frac{y+4}{2} = -3$$

$$y+4 = -6 \Rightarrow y = -10$$

Hence, coordinates of point A is $(1,-10)$.

63. Find a relation between x and y such that the point $P(x,y)$ is equidistant from the points $A(-5,3)$ and $B(7,2)$.

Ans : [Board Term-2 SQP 2016]

Let $P(x,y)$ is equidistant from $A(-5,3)$ and $B(7,2)$, then we have

$$AP = BP$$

$$\sqrt{(x+5)^2+(y-3)^2} = \sqrt{(x-7)^2+(y-2)^2}$$

$$(x+5)^2+(y-3)^2 = (x-7)^2+(y-2)^2$$

$$10x+25-6y+9 = -14x+49-4y+4$$

$$24x+34 = 2y+53$$

$$24x-2y = 19$$

Thus $24x-2y-19 = 0$ is the required relation.

64. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2,-5)$ and $R(-3,6)$, find the co-ordinates of P.

Ans : [Board Term-2 2016]

Let the point P be $(2y,y)$. Since $PQ = PR$, we have

$$\sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$$

$$(2y-2)^2+(y+5)^2 = (2y+3)^2+(y-6)^2$$

$$-8y+4+10y+25 = 12y+9-12y+36$$

$$2y+29 = 45$$

$$y = 8$$

Hence, coordinates of point P are $(16,8)$

65. Find the ratio in which y-axis divides the line segment joining the points $A(5,-6)$ and $B(-1,-4)$. Also find the co-ordinates of the point of division.

Ans : [Delhi Set I, II, III, 2016]

Let y -axis be divides the line-segment joining $A(5,-6)$ and $B(-1,-4)$ at the point $P(x,y)$ in the ratio $AP:PB = k:1$

Now, the coordinates of point of division P,

$$(x,y) = \left(\frac{k(-1)+1(5)}{k+1}, \frac{k(-4)+1(-6)}{k+1}\right)$$

$$= \left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$$

Since P lies on y axis, therefore $x = 0$, which gives

$$\frac{5-k}{k+1} = 0 \Rightarrow k = 5$$

Hence required ratio is 5:1,

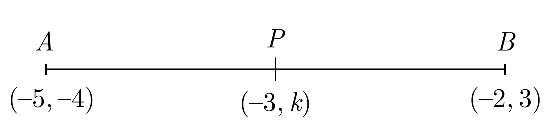
Now $$y = \frac{-4(5)-6}{6} = \frac{-13}{3}$$

Hence point on y-axis is $0, -\frac{13}{3}$.

66. Find the ratio in which the point $(-3, k)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Also find the value of k.

Ans : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



Let AB be divides by P in ratio n:1.

x co-ordinate for section formula

$$-3 = \frac{(-2)n+1(-5}{n+1}$$

$$-3(n+1) = -2n-5$$

$$-3n-3 = -2n-5$$

$$5-3 = 3n-2n$$

$$2 = n$$

Ratio $$\frac{n}{1} = \frac{2}{1} \text{ or } 2:1$$

Now, y co-ordinate,

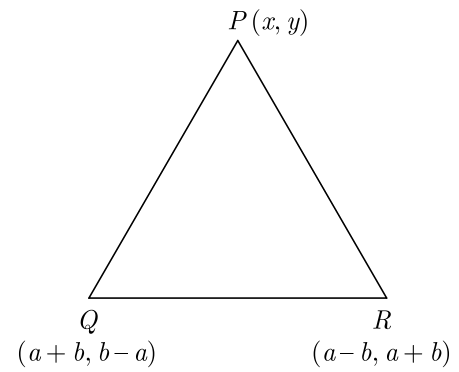
$$k = \frac{2(3)+1(-4)}{2+1} = \frac{6-4}{3} = \frac{2}{3}$$

67. If the point $P(x,y)$ is equidistant from the points $Q(a+b, b-a)$ and $R(a-b, a+b)$, then prove that $bx = ay$.

Ans : [Board Term-2 Delhi 2012, OD 2016]

We have $$|PQ| = |PR|$$

$$\sqrt{[x-(a+b)]^2+[y-(b-a)]^2} = \sqrt{[x-(a-b)]^2+[y-(b+a)]^2}$$



$$[x-(a+b)]^2+[y-(b-a)]^2 = [x-(a-b)]^2+[y-(a+b)]^2$$

$$-2x(a+b)-2y(b-a) = -2x(a-b)-2y(a+b)$$

$$2x(a+b)+2y(b-a) = 2x(a-b)+2y(a+b)$$

$$2x(a+b-a+b)+2y(b-a-a-b) = 0$$

$$2x(2b)+2y(-2a) = 0$$

$$xb-ay = 0$$

$$bx = ay$$ Hence Proved

68. Prove that the point (3,0), (6,4) and $(-1,3)$ are the vertices of a right angled isosceles triangle.

Ans :

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now $$AB^2 = (3-6)^2+(0-4)^2$$

g1

$$= 9 + 16 = 25$$

$$BC^2 = (6+1)^2 + (4-3)^2$$

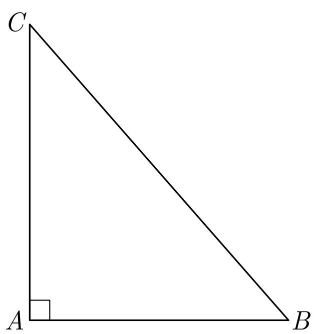
$$= 49 + 1 = 50$$

$$CA^2 = (-1-3)^2 + (3-0)^2$$

$$= 16 + 9 = 25$$

$$AB^2 = CA^2 \text{ or, } AB = CA$$

Hence triangle is isosceles.



Also, $25 + 25 = 50$

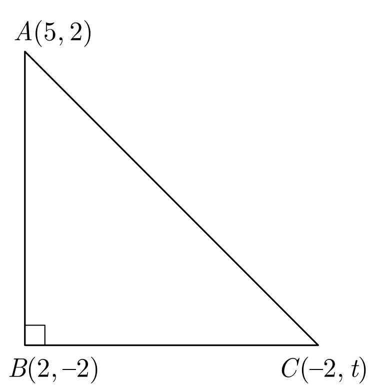
or, $AB^2 + CA^2 = BC^2$

Since Pythagoras theorem is verified, therefore triangle is a right angled triangle.

69. If $A(5,2)$, $B(2,-2)$ and $C(-2,t)$ are the vertices of a right angled triangle with $\angle B = 90^\circ$, then find the value of t.

Ans : [Board Term-2 Delhi 2015]

As per question, triangle is shown below.



Now $AB^2 = (2-5)^2 + (-2-2)^2 = 9 + 16 = 25$

$$BC^2 = (-2-2)^2 + (t+2)^2 = 16 + (t+2)^2$$

$$AC^2 = (5+2)^2 + (2-t)^2 = 49 + (2-t^2)$$

Since ΔABC is a right angled triangle

$$AC^2 = AB^2 + BC^2$$

$$49 + (2-t)^2 = 25 + 16 + (t+2)^2$$

$$49 + 4 - 4t + t^2 = 41 + t^2 + 4t + 4$$

$$53 - 4t = 45 + 4t$$

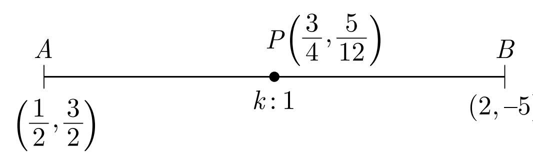
$$8t = 8$$

$$t = 1$$

70. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the point $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and $(2,-5)$.

Ans : [Board Term-2 Delhi 2015]

Let P divides AB in the ratio $k:1$. Line diagram is shown below.



Now $$\frac{k(2) + 1\left(\frac{1}{2}\right)}{k+1} = \frac{3}{4}$$

$$8k + 2 = 3k + 3$$

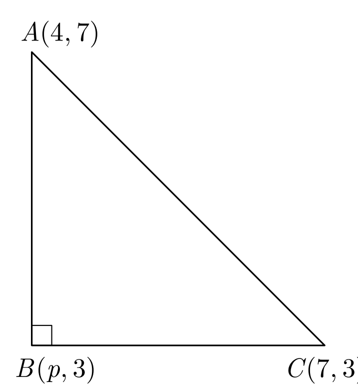
$$k = \frac{1}{5}$$

Thus required ratio is $\frac{1}{5}:1$ or 1:5.

71. The points $A(4,7)$, $B(p,3)$ and $C(7,3)$ are the vertices of a right triangle, right-angled at B. Find the value of p.

Ans : [Board Term-2 OD 2015]

As per question, triangle is shown below. Here ΔABC is a right angle triangle,



$$AB^2 + BC^2 = AC^2$$

$$(p-4)^2 + (3-7)^2 + (7-p)^2 + (3-3)^2 = (7-4)^2 + (3-4)^2$$

g1

$$(p-4)^2 + (-4)^2 + (7-p)^2 + 0 = (3)^2 + (-4)^2$$

$$p^2 - 8p + 16 + 16 + 49 + p^2 - 14p = 9 + 16$$

$$2p^2 - 22p + 81 = 25$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

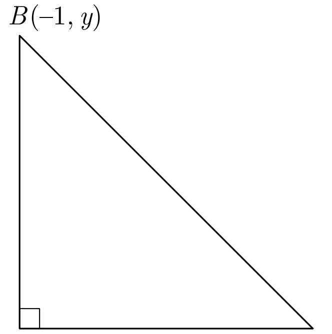
$$(p-4)(p-7) = 0$$

$$p = 7 \text{ or } 4$$

72. If $A(4,3)$, $B(-1,y)$, and $C(3,4)$ are the vertices of a right triangle ABC, right angled at A, then find the value of y.

Ans : [Board Term-2 OD 2015]

As per question, triangle is shown below.



Now $$AB^2 + AC^2 = BC^2$$

$$(4+1)^2 + (3-y)^2 + (4-3)^2 = (3+1)^2 + (4-y)^2$$

$$(5)^2 + (3-y)^2 + (-1)^2 + (1)^2 = (4)^2 + (4-y)^2$$

$$25 + 9 - 6y + y^2 + 1 + 1 = 16 + 16 - 8y + y^2$$

$$36 + 2y - 32 = 0$$

$$2y + 4 = 0$$

$$y = -2$$

73. Show that the points (a,a), $(-a,-a)$ and $(-\sqrt{3}\,a, \sqrt{3}\,a)$ are the vertices of an equilateral triangle.

Ans : [Board Term-2 Foreign 2015]

Let $A(a,a), B(-a,-a)$ and $C(-\sqrt{3}\,a, \sqrt{3}\,a)$.

Now $$AB = \sqrt{(a+a)^2 + (a+a)^2}$$

$$= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}\,a$$

g125

$$BC = \sqrt{(-a+\sqrt{3}\,a)^2 + (-a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 - 2\sqrt{3}\,a^2 + 3a^2 + a^2 + 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

$$AC = \sqrt{(a+\sqrt{3}\,a)^2 + (a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 + 2\sqrt{3}\,a^2 + 3a^2 + a^2 - 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

74. If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, find x, y.

Ans : [Board Term-2 OD 2012, Delhi 2014]

If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, then at mid point,

$$\frac{\frac{x}{2} + x + 1}{2} = 5$$

$$\frac{3x}{2} + 1 = 10$$

$$3x = 18 \Rightarrow x = 6$$

also $$\frac{\frac{y+1}{2} + y - 3}{2} = -2$$

$$\frac{y+1}{2} + y - 3 = -4$$

$$y + 1 + 2y - 6 = -8 \Rightarrow y = -1$$

75. Find the point on the x-axis which is equidistant from the points $(2,-5)$ and $(-2,9)$.

Ans : [Board Term-2 2012]

Let the point be $P(x,0)$ on the x-axis is equidistant from points $A(2,-5)$ and $B(-2,9)$.

Now $$PA^2 = PB^2$$

$$(2-x)^2 + (-5-0)^2 = (-2-x)^2 + (9-0)^2$$

$$4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$$

$$-8x = 56 \Rightarrow x = -7$$

Thus point is $(-7,0)$.

76. Show that $A(6,4)$, $B(5,-2)$ and $C(7,-2)$ are the vertices of an isosceles triangle.

Ans : [Board Term-2, 2012]

We have $A(6,4), B(5,-2), C(7,-2)$.

Now $AB = \sqrt{(6-5)^2+(4+2)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$BC = \sqrt{(5-7)^2+(-2+2)^2}$

$= \sqrt{(-2)^2+0^2} = 2$

$CA = \sqrt{(7-6)^2+(-2-4)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$AB = BC = \sqrt{37}$

Since two sides of a triangle are equal in length, triangle is an isosceles triangle.

77. If $P(2,-1)$, $Q(3,4)$, $R(-2,3)$ and $S(-3,-2)$ be four points in a plane, show that $PQRS$ is a rhombus but not a square.

Ans : [Board Term-2 OD 2012]

We have $P(2,-1)$, $Q(3,4)$, $R(-2,3)$, $S(-3,-2)$

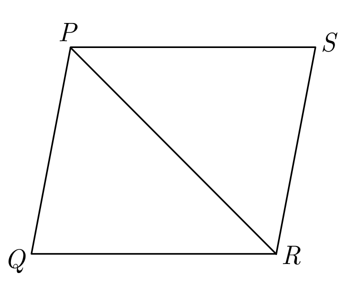
$PQ = \sqrt{1^2+5^2} = \sqrt{26}$

$QR = \sqrt{5^2+1^2} = \sqrt{26}$

$RS = \sqrt{1^2+5^2} = \sqrt{26}$

$PS = \sqrt{5^2+1^2} = \sqrt{26}$

Since all the four sides are equal, $PQRS$ is a rhombus.



Now $PR = \sqrt{1^2+5^2} = \sqrt{26}$

$= \sqrt{4^2+4^2} = \sqrt{32}$

$PQ^2+QR^2 = 2\times 26 = 52 \neq (\sqrt{32})^2$

Since ΔPQR is not a right triangle, $PQRS$ is a rhombus but not a square.

78. Show that $A(-1,0)$, $B(3,1)$, $C(2,2)$ and $D(-2,1)$ are the vertices of a parallelogram $ABCD$.

Ans : [Board Term-2 2012]

Mid-point of AC,

$$\left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2},1\right)$$

Mid-point of BD,

$$\left(\frac{3-2}{2}, \frac{1+1}{2}\right) = \left(\frac{1}{2},1\right)$$

Here Mid-point of AC = Mid-point of BD

Since diagonals of a quadrilateral bisect each other, $ABCD$ is a parallelogram.

79. If $(3,2)$ and $(-3,2)$ are two vertices of an equilateral triangle which contains the origin, find the third vertex.

Ans : [Board Term-2 OD 2012]

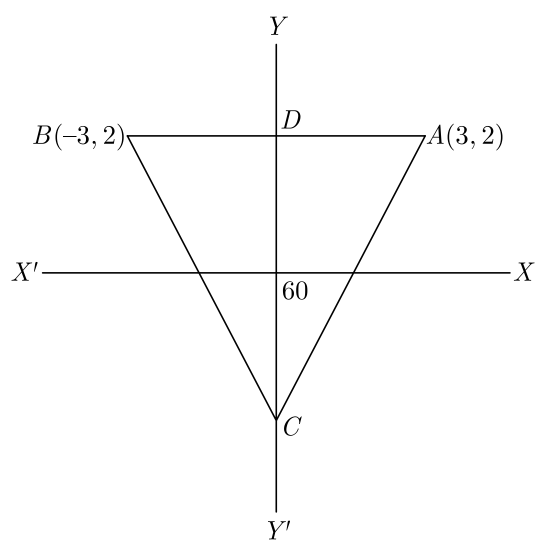
We have $A(3,2)$ and $B(-3,2)$.

It can be easily seen that mid-point of AB is lying on y-axis. Thus AB is equal distance from x-axis everywhere.

Also $OD \perp AB$

Hence 3^{rd} vertex of ΔABC is also lying on y-axis.

The digram of triangle should be as given below.



Let $C(x,y)$ be the coordinate of 3^{rd} vertex of ΔABC.

Now $AB^2 = (3+3)^2+(2-2)^2 = 36$

$BC^2 = (x+3)^2+(y-2)^2$

$AC^2 = (x-3)^2+(y-2)^2$

Since $AB^2 = AC^2 = BC^2$

$(x+3)^2+(y-2)^2 = 36 \quad (1)$

$(x-3)^2+(y-2)^2 = 36 \quad (2)$

Since $P(x,y)$ lie on y-axis, substituting $x=0$ in (1) we have

$$3^2 + (y-2)^2 = 36 - 9 = 27$$
$$(y-2)^2 = 36 - 9 = 27$$

Taking square root both side

$$y - 2 = \pm 3\sqrt{3}$$
$$y = 2 \pm 3\sqrt{3}$$

Since origin is inside the given triangle, coordinate of C below the origin,

$$y = 2 - 3\sqrt{3}$$

Hence Coordinate of C is $\left(0,\ 2-3\sqrt{3}\right)$

80. Find a so that $(3, a)$ lies on the line represented by $2x - 3y - 5 = 0$. Also, find the co-ordinates of the point where the line cuts the x-axis.

Ans : [Board Term-2 2012]

Since $(3, a)$ lies on $2x - 3y - 5 = 0$, it must satisfy this equation. Therefore

$$2 \times 3 - 3a - 5 = 0$$
$$6 - 3a - 5 = 0$$
$$1 = 3a$$
$$a = \frac{1}{3}$$

Line $2x - 3y - 5 = 0$ will cut the x-axis at $(x, 0)$. and it must satisfy the equation of line.

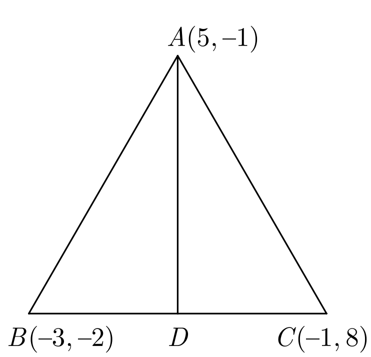
$$2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

Hence point is $\left(\frac{5}{2}, 0\right)$.

81. If the vertices of ΔABC are $A(5, -1)$, $B(-3, -2)$, $C(-1, 8)$, Find the length of median through A.

Ans : [Board Term-2 2012]

Let AD be the median. As per question, triangle is shown below.



Since D is mid-point of BC, co-ordinates of D,

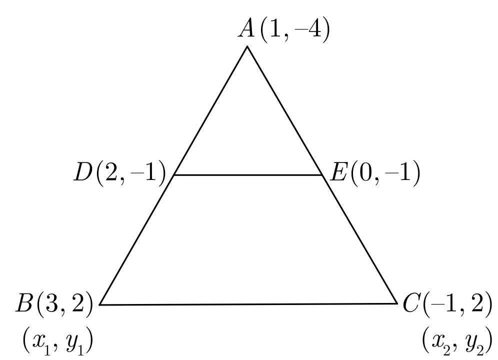
$$(x_1,\ y_2) = \left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$$
$$= (-2,\ 3)$$
$$AD = \sqrt{(5+2)^2 + (-1-3)^2}$$ g133
$$= \sqrt{(7)^2 + (4)^2}$$
$$= \sqrt{49+16} = \sqrt{65} \text{ units}$$

Thus length of median is $\sqrt{65}$ units.

82. Find the mid-point of side BC of ΔABC, with $A(1, -4)$ and the mid-points of the sides through A being $(2, -1)$ and $(0, -1)$.

Ans : [Board Term-2 2012]

Assume co-ordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively. As per question, triangle is shown below.



Now $$2 = \frac{1+x_1}{2} \Rightarrow x_1 = 3$$

and $$-1 = \frac{-4+y_1}{2} \Rightarrow y_1 = 2$$
$$0 = \frac{1+x_2}{2} \Rightarrow x = -1$$
$$-1 = \frac{-4+y_2}{2} \Rightarrow y_2 = 2$$

Thus $$B(x_1, y_1) = (3, 2),$$
$$C(x_2, y_2) = (-1, 2)$$

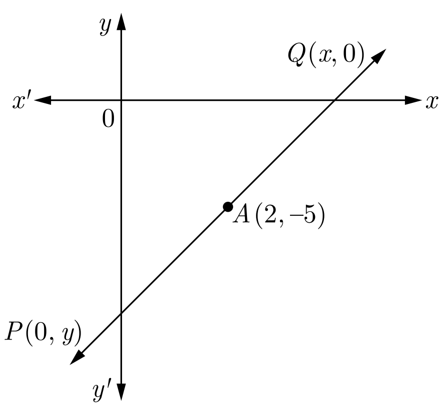
So, mid-point of BC is $\left(\frac{3-1}{2}, \frac{2+2}{2}\right) = (1, 2)$

83. A line intersects the y-axis and x-axis at the points P

and Q respectively. If $(2,-5)$ is the mid-point of PQ, then find the coordinates of P and Q.

Ans : [Board Term-2 OD 2017]

Let coordinates of P be $(0, y)$ and of Q be $(x, 0)$.
$A(2,-5)$ is mid point of PQ.
As per question, line diagram is shown below.



Using section formula,

$$(2,-5) = \left(\frac{0+x}{2} + \frac{y+0}{2}\right)$$

$$2 = \frac{x}{2} \Rightarrow x = 4$$

and $$-5 = \frac{y}{2} \Rightarrow y = -10$$

Thus P is $(0,-10)$ and Q is $(4,0)$

84. If $(1, \frac{p}{3})$ is the mid point of the line segment joining the points $(2,0)$ and $(0, \frac{2}{9})$, then show that the line $5x+3y+2=0$ passes through the point $(-1, 3p)$.

Ans :

Since $(1, \frac{p}{3})$ is the mid point of the line segment joining the points $(2,0)$ and $(0, \frac{2}{9})$, we have

$$\frac{p}{3} = \frac{0+\frac{2}{9}}{2} = \frac{1}{9}$$

$$p = \frac{1}{3}$$

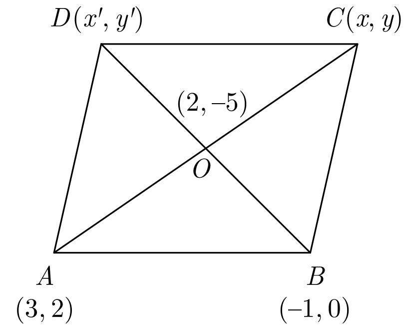
Now the point $(-1, 3p)$ is $(-1, 1)$.
The line $5x+3y+2=0$, passes through the point $(-1,1)$ as $5(-5)+3(1)+2=0$

85. If two adjacent vertices of a parallelogram are $(3,2)$ and $(-1,0)$ and the diagonals intersect at $(2,-5)$ then find the co-ordinates of the other two vertices.

Ans : [Board Term-2 Foreign 2017]

Let two other co-ordinates be (x, y) and $(x^{'}, y^{'})$ respectively using mid-point formula.

As per question parallelogram is shown below.



Now $$2 = \frac{x+3}{2} \Rightarrow x = 1$$

and $$-5 = \frac{2+y}{2} \Rightarrow y = -12$$

Again, $$\frac{-1+x^{'}}{2} = 2 \Rightarrow x^{'} = 5$$

and $$\frac{0+y^{'}}{2} = -5 \Rightarrow y^{'} = -10$$

Hence, coordinates of $C(1,-12)$ and $D(5,-10)$

86. In what ratio does the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$ and $B(3,-8)$?

Ans : [Board Term-2 Delhi Compt. 2017]

Let $$AP:PB = k:1$$

Now $$\frac{3k-6}{k+1} = -4$$

$$3k-6 = -4k-4$$

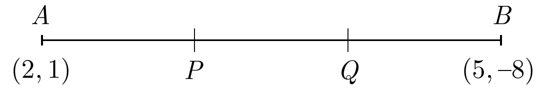
$$7k = 2 \Rightarrow k = \frac{2}{7}$$

Hence, $$AP:PB = 2:7$$

87. If the line segment joining the points $A(2,1)$ and $B(5,-8)$ is trisected at the points P and Q, find the coordinates P.

Ans : [Board Term-2 OD Compt. 2017]

As per question, line diagram is shown below.



Let $P(x, y)$ divides AB in the ratio 1:2
Using section formula we get

$$x = \frac{1 \times 5 + 2 \times 2}{1+2} = 3$$

$$y = \frac{1 \times -8 + 2 \times 1}{1+2} = -2$$

Hence coordinates of P are $(3, -2)$.

88. Prove that the points $(2,-2), (-2,1)$ and $(5,2)$ are the vertices of a right angled triangle. Also find the area of this triangle.

Ans : [Board Term-2 Foreign 2016]

We have $A(2,-2), B(-2,1)$ and $(5,2)$

Now using distance formula we get

$$AB^2 = (2+2)^2 + (-2-1)^2$$

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$$= 16 + 9 = 25$$

$$AB^2 = 25 \Rightarrow AB = 5.$$

Thus $AB = 5$.

Similarly $$BC^2 = (-2-5)^2 + (1-2)^2$$

$$= 49 + 1 = 50$$

$$BC^2 = 50 \Rightarrow BC = 5\sqrt{2}$$

$$AC^2 = (2-5)^2 + (-2-2)^2$$

$$= 9 + 16 = 25$$

$$AC^2 = 25 \Rightarrow AC = 5$$

Clearly $$AB^2 + AC^2 = BC^2$$

$$25 + 25 = 50$$

Hence the triangle is right angled,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$

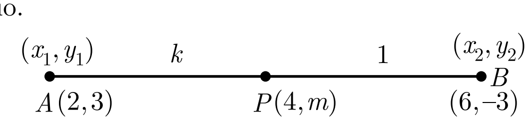
$$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2} \text{ sq unit.}$$

THREE MARKS QUESTIONS

89. Find the ratio in which $P(4,m)$ divides the segment joining the points $A(2,3)$ and $B(6,-3)$. Hence find m.

Ans : [Board 2018]

Let $P(x,y)$ be the point which divide AB in $k:1$ ratio.



Now $$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$4 = \frac{k(6) + 1(2)}{k+1}$$

$$4k + 4 = 6k + 2$$

$$6k - 4k = 4 - 2$$

$$2k = 2 \Rightarrow k = 1$$

Thus point P divides the line segment AB in $1:1$ ratio.

Now $$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$m = \frac{1 \times (-3) + 1(3)}{1+1}$$

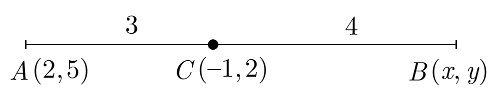
$$= \frac{-3+3}{2} = 0$$

Thus $m = 0$.

90. If the point $C(-1,2)$ divides internally the line segment joining $A(2,5)$ and $B(x,y)$ in the ratio $3:4$ find the coordinates of B.

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



Using section formula,

$$-1 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$-1 = \frac{3 \times x + 4 \times 2}{3+4} = \frac{3x+8}{7}$$

$$3x + 8 = -7$$

$$3x = -15 \Rightarrow x = -5$$

and $$2 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$2 = \frac{3y + 4 \times 5}{3+4} = \frac{3y+20}{7}$$

$$3y + 20 = 14$$

$$3y = 14 - 20 = -6$$

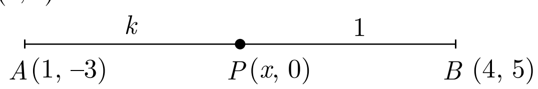
$$y = -2$$

Hence, the coordinates of $B(x, y)$ is $(-5, -2)$.

91. Find the ratio in which the segment joining the points $(1, -3)$ and $(4, 5)$ is divided by x-axis? Also find the coordinates of this point on x-axis.

Ans : [Board 2019 Delhi]

Let the required ratio be $k:1$ and the point on x-axis be $(x, 0)$.



Here, $(x_1, y_1) = (1, -3)$

and $(x_2, y_2) = (4, 5)$

Using section formula y coordinate, we obtain,

$$y = \frac{my_2 + ny_1}{m+n}$$

$$0 = \frac{k \times 5 + 1 \times 1(\quad 3}{k+1}$$

$$0 = 5k - 3$$

$$5k = 3 \Rightarrow k = \frac{3}{5}$$

Hence, the required ratio is $\frac{3}{5}$ i.e $3:5$.

Now, again using section formula for x, we obtain

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$x = \frac{k \times (4) + 1 \times 1}{k+1}$$

$$= \frac{\frac{3}{5}(4) + 1}{\frac{3}{5} + 1} = \frac{12+5}{3+5} = \frac{17}{8}$$

Co-ordinate of P is $\left(\frac{17}{8}, 0\right)$.

92. Find the point on y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$.

Ans : [Board 2019 Delhi]

We have point $A = (5, -2)$ and $B = (-3, 2)$

Let $C(0, a)$ be point on y-axis.

According to question, point C is equidistant from A and B.

Thus $AC = BC$

Using distance formula we have

$$\sqrt{(0-5)^2 + (a+2)^2} = \sqrt{(0+3)^2 + (a-2)^2}$$

$$\sqrt{25 + a^2 + 4 + 4a} = \sqrt{9 + a^2 + 4 - 4a}$$

$$25 + a^2 + 4 + 4a = 9 + a^2 + 4 - 4a$$

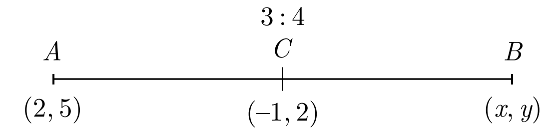
$$8a = -16 \Rightarrow a = -2$$

Hence, point on y-axis is $(0 - 2)$.

93. If the point $C(-1, 2)$ divides internally the line segment joining the points $A(2, 5)$ and $B(x, y)$ in the ratio $3:4$, find the value of $x^2 + y^2$.

Ans : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



We have $\frac{AC}{BC} = \frac{3}{4}$

Applying section formula for x co-ordinate,

$$-1 = \frac{3x + 4(2)}{3+4}$$

$$-7 = 3x + 8 \Rightarrow x = -5$$

Similarly applying section formula for y co-ordinate,

$$2 = \frac{3y + 4(5)}{3+4}$$

$$14 = 3y + 20 \Rightarrow y = -2$$

Thus (x, y) is $(-5, -2)$.

Now $x^2 + y^2 = (-5)^2 + (-2)^2$

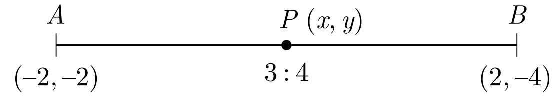
$$= 25 + 4 = 29$$

94. If the co-ordinates of points A and B are $(-2, -2)$ and $(2, -4)$ respectively, find the co-ordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB.

Ans : [Board Term-2 OD 2017]

We have $AP = \frac{3}{7}AB \Rightarrow AP:PB = 3:4$

As per question, line diagram is shown below.



Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

Applying section formula we get

$$x = \frac{3 \times 2 + 4 \times (-2)}{3+4} = -\frac{2}{7}$$

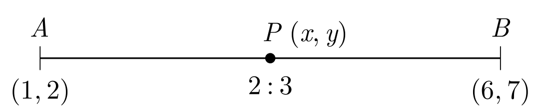
$$y = \frac{3 \times (-4) + 4 \times (-2)}{3+4} = -\frac{20}{7}$$

Hence P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

95. Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$.

Ans : [Board Term-2 OD 2015]

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP{:}PB = 2{:}3$

Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2 \times 6 + 3 \times 1}{2+3} = \frac{12+3}{5} = 3$$

and $$y = \frac{2 \times 7 + 3 \times 2}{2+3} = \frac{14+6}{5} = 4$$

Thus $P(x,y) = (3,4)$

96. If the distance of $P(x,y)$ from $A(6,2)$ and $B(-2,6)$ are equal, prove that $y = 2x$.

Ans : [Board Term-2, 2015]

We have $P(x,y), A(6,2), B(-2,6)$

Now $PA = PB$

$$PA^2 = PB^2$$

$$(x-6)^2 + (y-2)^2 = (x+2)^2 + (y-$$

$$-12x + 36 - 4y + 4 = 4x + 4 - 12y + 36$$

$$-12x - 4y = 4x - 12y$$

$$12y - 4y = 4x + 12x$$

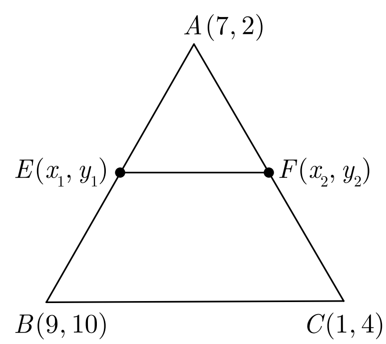
$$8y = 16x$$

$$y = 2x$$ Hence Proved

97. The co-ordinates of the vertices of ΔABC are $A(7,2)$, $B(9,10)$ and $C(1,4)$. If E and F are the mid-points of AB and AC respectively, prove that $EF = \frac{1}{2}BC$.

Ans : [Board Term-2 2015]

Let the mid-points of AB and AC be $E(x_1, y_1)$ and $F(x_2, y_2)$. As per question, triangle is shown below.



Co-ordinates of point E,

$$(x_1, y_1) = \left(\frac{9+7}{2}, \frac{10+2}{2}\right) = (8,6)$$

Co-ordinates of point F,

$$(x_2, y_2) = \left(\frac{7+1}{2}, \frac{2+4}{2}\right) = (4,3)$$

Length, $$EF = \sqrt{(8-4)^2 + (6-3)^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= 5 \text{ units} \quad ...(1)$$

Length $$BC = \sqrt{(9-1)^2 + (10-4)^2}$$

$$= \sqrt{8^2 + 6^2}$$

$$= 10 \text{ units} \quad ...(2)$$

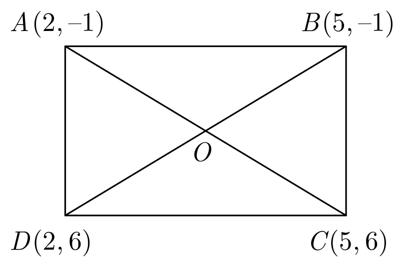
From equation (1) and (2) we get

$$EF = \frac{1}{2}BC$$ Hence proved.

98. Prove that the diagonals of a rectangle $ABCD$, with vertices $A(2, -1), B(5, -1), C(5, 6)$ and $D(2,6)$ are equal and bisect each other.

Ans : [Board Term-2 2014]

As per question, rectangle $ABCD$, is shown below.



Now
$$AC = \sqrt{(5-2)^2+(6+1)^2}$$
$$= \sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$$
$$BD = \sqrt{(5-2)^2+(-1-6^{\ 2}}$$
$$= \sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$$

Since $AC = BD = \sqrt{58}$ the diagonals of rectangle $ABCD$ are equal.

Mid-point of AC,
$$= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

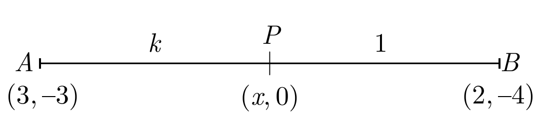
Mid-point of BD,
$$= \left(\frac{2+5}{2}, \frac{6-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

Since the mid-point of diagonal AC and mid-point of diagonal BD is same and equal to $\left(\frac{7}{5}, \frac{5}{2}\right)$. Hence they bisect each other.

99. Find the ratio in which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of point of division.

Ans : [Board Term-2 Delhi 2014]

We know that y co-ordinate of any point on the $x-$ axis will be zero. Let $(x,\ 0)$ be point on x axis which cut the line. As per question, line diagram is shown below.



Let the ratio be $k:1$. Using section formula for y co-ordinate we have
$$0 = \frac{1(-3)+k(7)}{1+k}$$
$$k = \frac{3}{7}$$

Using section formula for x co-ordinate we have
$$x = \frac{1(3)+k(-2)}{1+k} = \frac{3-2\times\frac{3}{7}}{1+\frac{3}{7}} = \frac{3}{2}$$

Thus co-ordinates of point are $\left(\frac{3}{2}, 0\right)$.

100. Find the ratio in which $(11,15)$ divides the line segment joining the points $(15,5)$ and $(9,20)$.

Ans : [Board Term-2 2014]

Let the two points $(15,5)$ and $(9,20)$ are divided in the ratio $k:1$ by point $P(11,15)$.
Using Section formula, we get
$$x = \frac{m_2x_1+m_1x_2}{m_2+m_1}$$
$$11 = \frac{1(15)+k(9)}{1+k}$$
$$11+11k = 15+9k$$
$$k = 2$$

Thus ratio is 2 : 1.

101. Find the point on y-axis which is equidistant from the points $(5,-2)$ and $(-3,2)$.

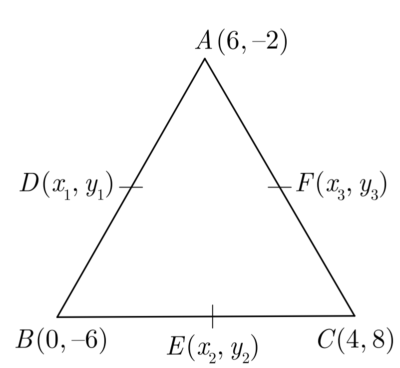
Ans : [Board Term-2 2014, Delhi 2012]

Let point be $(0,y)$.
$$5^2+(y+2)^2 = (3)^2+(y-2)^2$$
or, $$y^2+25+4y+4 = 9-4y+4$$
$$8y = -16 \text{ or, } y = -2$$
or, Point $(0,-2)$

102. The vertices of ΔABC are $A(6,-2)$, $B(0,-6)$ and $C(4,8)$. Find the co-ordinates of mid-points of AB, BC and AC.

Ans : [Board Term-2, 2014]

Let mid-point of AB, BC and AC be $D(x_1,y_1)$, $E(x_2,y_2)$ and $F(x_2,y_3)$. As per question, triangle is shown below.



Using section formula, the co-ordinates of the points D, E, F are

For D, $$x_1 = \frac{6+0}{2} = 3$$

$$y_1 = \frac{-2-6}{2} = -4$$

For E, $$x_2 = \frac{0+4}{2} = 2$$

$$y_2 = \frac{-6+8}{2} = 1$$

For F, $$x_3 = \frac{4+6}{2} = 5$$

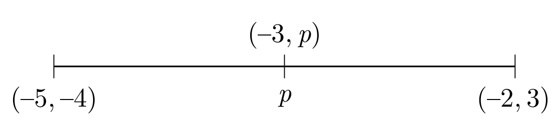
$$y_3 = \frac{-2+8}{2} = 3$$

The co-ordinates of the mid-points of AB, BC and AC are $D(3,-4)$, $E(2,1)$ and $F(5,3)$ respectively.

103. Find the ratio in which the point $(-3,p)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Hence find the value of p.

Ans : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let $X(-3,p)$ divides the line joining of $A(-5,-4)$ and $B(-2,3)$ in the ratio $k:1$.

The co-ordinates of p are $\left[\frac{-2k-5}{k+1}, \frac{3k-4}{k+1}\right]$

But co-ordinates of P are $(-3,p)$. Therefore we get

$$\frac{-2k-5}{k+1} = -3 \Rightarrow k = 2$$

and $$\frac{3k-4}{k+1} = p$$

Substituting $k = 2$ gives

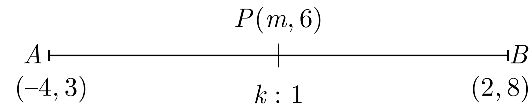
$$p = \frac{2}{3}$$

Hence ratio of division is 2:1 and $p = \frac{2}{3}$

104. Find the ratio in which the point $p(m,6)$ divides the line segment joining the points $A(-4,3)$ and $B(2,8)$. Also find the value of m.

Ans : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let the ratio be $k:1$.

Using section formula, we have

$$m = \frac{2k+(-4)}{k+1} \quad (1)$$

$$6 = \frac{8k+3}{k+1} \quad (2)$$

$$8k+3 = 6k+6$$

$$2k = 3 \Rightarrow k = \frac{3}{2}$$

Thus ratio is $\frac{3}{2}:1$ or $3:2$.

Substituting value of k in (1) we have

$$m = \frac{2\left(\frac{3}{2}\right)+(-4)}{\frac{3}{2}+1} = \frac{3-4}{\frac{5}{2}} = \frac{-1}{\frac{5}{2}} = \frac{-2}{5}$$

105. If $A(4,-1), B(5,3), C(2,y)$ and $D(1,1)$ are the vertices of a parallelogram $ABCD$, find y.

Ans : [Board Term-2, 2012]

Diagonals of a parallelogram bisect each other.

Mid-points of AC and BD are same.

Thus $$\left(3, \frac{-1+y}{2}\right) = (3,2)$$

$$\frac{-1+y}{2} = 2 \Rightarrow y = 5$$

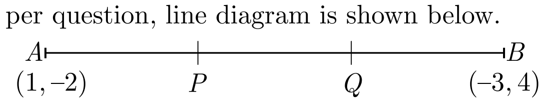
106. Find the co-ordinates of the points of trisection of the line segment joining the points $A(1,-2)$ and $B(-3,4)$.

Ans : [Board Term-2, 2012]

Let $P(x_1,y_1), Q(x_2,y_2)$ divides AB into 3 equal parts.

Thus P divides AB in the ratio of 1:2.

As per question, line diagram is shown below.



Now $$x_1 = \frac{1(-3)+2(1)}{1+3} = \frac{-3+2}{3} = \frac{-1}{3}$$

$$y_1 = \frac{1(4)+2(-2)}{1+2} = \frac{4-4}{3} = 0$$

Co-ordinates of P is $(-\frac{1}{3}, 0)$.

Here Q is mid-point of PB.

Thus $$x_2 = \frac{-\frac{1}{3}+(-3)}{2} = \frac{-10}{6} = \frac{-5}{3}$$

$$y_2 = \frac{0+4}{2} = 2$$

Thus co-ordinates of Q is $(-\frac{5}{2}, 2)$.

107. If (a, b) is the mid-point of the segment joining the points $A(10, -6)$ and $B(k, 4)$ and $a - 2b = 18$, find the value of k and the distance AB.

Ans : [Board Term-2, 2012]

We have $A(10, -6)$ and $B(k, 4)$.
If $P(a, b)$ is mid-point of AB, then we have

$$(a, b) = \left(\frac{k+10}{2}, \frac{-6+4}{2}\right)$$

$$a = \frac{k+10}{2} \text{ and } b = -1$$

From given condition we have

$$a - 2b = 18$$

Substituting value $b = -1$ we obtain

$$a + 2 = 18 \Rightarrow a = 16$$

$$a = \frac{k+10}{2} = 16 \Rightarrow k = 22$$

$$P(a, b) = (16, 1)$$

$$AB = \sqrt{(22-10)^2 + (4+6)^2}$$

$$= 2\sqrt{61} \text{ units}$$

108. Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points $(8, -9)$ and $(2, 1)$. Also find the co-ordinates of the point of division.

Ans : [Board Term-2, 2012]

Let a point $P(x, y)$ on line $2x + 3y - 5 = 0$ divides AB in the ratio $k:1$.

Now $$x = \frac{2k+8}{k+1}$$

and $$y = \frac{k-9}{k+1}$$

Substituting above value in line $2x + 3y - 5 = 0$ we have

$$2\left(\frac{2k+8}{k+1}\right) + 3\left(\frac{k-9}{k+1}\right) - 5 = 0$$

$$4k + 16 + 3k - 27 - 5k - 5 = 0$$

$$2k - 16 = 0$$

$$k = 8$$

Thus ratio is 8 : 1.

Substituting the value $k = 8$ we get

$$x = \left(\frac{2 \times 8 + 8}{8+1}\right) = \frac{8}{3}$$

$$y = \left(\frac{8-9}{8+1}\right) = -\frac{1}{9}$$

Thus $$P(x, y) = \left(\frac{8}{3}, -\frac{1}{9}\right)$$

109. Find the area of the rhombus of vertices $(3, 0), (4, 5), (-1, 4)$ and $(-2, -1)$ taken in order.

Ans : [Board Term-2, 2012]

We have $A(3, 0)$, $B(4, 5)$, $C(-1, 4)$, $D(-2, -1)$

Diagonal AC, $$d_1 = \sqrt{(3+1)^2 + (0-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32}$$
$$= \sqrt{16 \times 2} = 4\sqrt{2}$$

Diagonal BD, $$d_2 = \sqrt{(4+2)^2 + (5+1)^2}$$
$$= \sqrt{36+36} = \sqrt{72}$$
$$= \sqrt{36 \times 2} = 6\sqrt{2}$$

Area of rhombus $$= \frac{1}{2} \times d_1 \times d_2$$
$$= \frac{1}{2} 4\sqrt{2} \times 6\sqrt{2}$$
$$= 24 \text{ sq. unit.}$$

110. Find the ratio in which the line joining points $(a+b, b+a)$ and $(a-b, b-a)$ is divided by the point (a, b).

Ans : [Board Term-2, 2013]

Let $A(a+b, b+a)$, $B(a-b, b-a)$ and $P(a, b)$ and P divides AB in $k:1$, then we have

$$a = \frac{k(a-b) + 1(a+b)}{k+1}$$

$$a(k+1) = k(a-b) + a + b$$

$$ak + a = ak - bk + a + b$$

$$bk = b$$

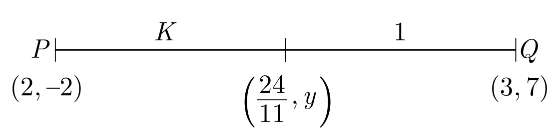
$$k = 1$$

Thus (a, b) divides $A(a+b, b+a)$ and $B(a-b, b-a)$ in 1:1 internally.

111. In what ratio does the point $(\frac{24}{11}, y)$ divides the line segment joining the points $P(2,-2)$ and $Q(3,7)$? Also find the value of y.

Ans : [Board Term-2 SQP 2012]

As per question, line diagram is shown below.



Let $P(\frac{24}{11}, y)$ divides the segment joining the points $P(2,-2)$ and $Q(3,7)$ in ratio $k:1$.

Using intersection formula $x = \frac{mx_2 + nx_1}{m+n}$ we have

$$\frac{3k+2}{k+1} = \frac{24}{11}$$

$$33k + 22 = 24k + 24$$

$$9k = 2 \Rightarrow k = \frac{2}{9}$$

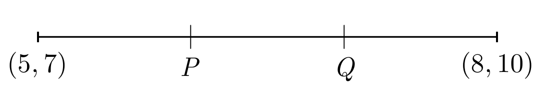
Hence, $$y = \frac{-18+14}{11} = -\frac{4}{11}$$

112. Find the co-ordinates of the points which divide the line segment joining the points $(5,7)$ and $(8,10)$ in 3 equal parts.

Ans : [Board Term-2 OD Compt. 2017]

Let $P(x_1, y_2)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.



Using section formula we have,

Now $$x = \frac{1(8)+2(5)}{3} = 6$$

$$y = \frac{1(10)+2(7)}{3} = 8$$

Thus $P(x_1, y_1)$ is $P(6,8)$. Since Q is the mid point of PB, we have

$$x_1 = \frac{6+8}{2} = 7$$

$$y_1 = \frac{8+10}{2} = 9$$

Thus $Q(x_2, y_2)$ is $Q(7,9)$

113. Find the co-ordinates of a point on the $x-$axis which is equidistant from the points $A(2,-5)$ and $B(-2,9)$.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the point P on the x axis be $(x,0)$. Since it is equidistant from the given points $A(2,-5)$ and $B(-2,9)$

$$PA = PB$$

$$PA^2 = PB^2$$

$$(x-2)^2 + [0-(-5)]^2 = (x-(-2))^2 + 0-9)^2$$

$$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$

$$-4x + 29 = 4x + 85$$

$$x = -\frac{56}{8} = -7$$

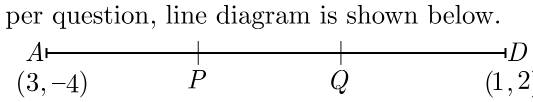
Hence the point on x axis is $(-7, 0)$

114. The line segment joining the points $A(3,-4)$ and $B(1,2)$ is trisected at the points P and Q. Find the coordinate of the PQ.

Ans : [Delhi Compt. Set-II, 2017]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2.

As per question, line diagram is shown below.



Using intersection formula

$$x = \frac{1 \times 1 + 2 \times 3}{1+2} = \frac{7}{3}$$

$$y = \frac{1 \times 2 + 2 \times -4}{1+2} = -2$$

Hence point P is $\left(\frac{7}{3}, -2\right)$

115. Show that ΔABC with vertices $A(-2,0), B(0,2)$ and $C(2,0)$ is similar to ΔDEF with vertices

$D(-4,0), F(4,0)$ and $E(0,4)$.

Ans : [Board Term-2 Delhi 2017, Foreign 2017]

Using distance formula

$$AB = \sqrt{(0+2)^2+(2-0)^2} = \sqrt{4+}$$
$$= 2\sqrt{2} \text{ units}$$
$$BC = \sqrt{(2-0)^2+(0-2)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$
$$CA = \sqrt{(-2-2)^2+(0-0)^2} = \sqrt{16}$$
$$= 4 \text{ units}$$

and $$DE = \sqrt{(0+4)^2+(4-0)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$EF = \sqrt{(4-0)^2+(0-4)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$FD = \sqrt{(-4-4)^2+(0-0)^2} = \sqrt{64}$$
$$= 8 \text{ units}$$
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
$$\frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{4}{8} = \frac{1}{2}$$

Since ratio of the corresponding sides of two similar Δs is equal, we have

$\Delta ABC \sim \Delta DEF$ Hence Proved.

116. Find the co-ordinates of the point on the $y-$ axis which is equidistant from the points $A(5,3)$ and $B(1,-5)$

Ans : [Board Term-2 OD Compt. 2017]

Let the points on y-axis be $P(0,y)$

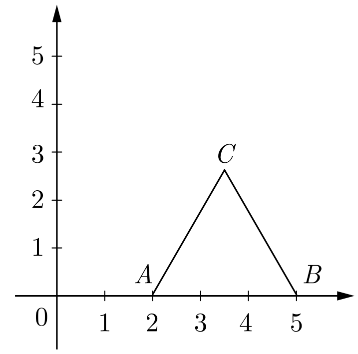
Now $$PA = PB$$
$$PA^2 = PB^2$$

g167

$$(0-5)^2+(y-3)^2 = (0-1)^2+(y+5)^2$$
$$5^2+y^2-6y+9 = 1+y^2+10y+25$$
$$16y = 8 \Rightarrow y = \frac{1}{2}$$

Hence point on y-axis is $(0,\frac{1}{2})$.

117. In the given figure ΔABC is an equilateral triangle of side 3 units. Find the co-ordinates of the other two vertices.



Ans : [Board Term-2 Foreign 2017]

The co-ordinates of B will be $(2+3,0)$ or .$(5,0)$

Let co-ordinates of C be (x,y). Since triangle is equilateral, we have

$$AC^2 = BC^2$$
$$(x-2)^2(y-0)^2 = (x-5)^2+(y-0)^2$$
$$x^2+4-4x+y^2 = x^2+25-10x+y^2$$

g168

$$6x = 21$$
$$x = \frac{7}{2}$$

and $$(x-2)^2+(y-0)^2 = 9$$
$$\left(\frac{7}{2}-2\right)^2+y^2 = 9$$
$$\frac{9}{4}+y^2 = 9 \text{ or, } y^2 = 9-\frac{9}{4}$$
$$y^2 = \frac{27}{4} = \frac{3\sqrt{3}}{2}$$

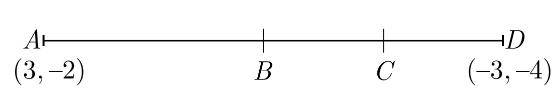
Hence C is $\left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$.

118. Find the co-ordinates of the points of trisection of the line segment joining the points $(3,-2)$ and $(-3,-4)$.

Ans : [Board Term-2 Foreign 2017]

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ trisect the line joining $A(3,-2)$ and $B(-3,-4)$.

As per question, line diagram is shown below.



Thus P divides AB in the ratio 1:2.

Using intersection formula $x = \frac{mx_2+nx_1}{m+n}$ and $y = \frac{my_2+ny_1}{m+n}$

$$x_1 = \frac{1(-3)+2(3)}{1+2} = 1$$

and $$y_1 = \frac{1(-4)+2(-2)}{1+2} = -\frac{8}{3}$$

Thus we have $x = 1$ and $y = -\frac{8}{3}$

Since Q is at the mid-point of PB, using mid-point formula

$$x_2 = \frac{1-3}{2} = -1$$

and $$y_2 = \frac{-\frac{8}{3}+(-4)}{2} = -\frac{10}{3}$$

Hence the co-ordinates of P and Q are $(1, -\frac{8}{3})$ and $(-1, -\frac{10}{3})$

119. If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$.

Ans : [Board Term-2 OD 2016]

Since $P(x, y)$ is equidistant from the given points $A(5, 1)$ and $B(-1, 5)$,

$$PA = PB$$

$$PA^2 = PB^2$$

Using distance formula,

$$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$$

$$(5-x)^2 + (1-y)^2 = (1+x)^2 + (5-y)^2$$

$$25 - 10x + 1 - 2y = 1 + 2x + 25 - 10y$$

$$-10x - 2y = 2x - 10y$$

$$8y = 12x$$

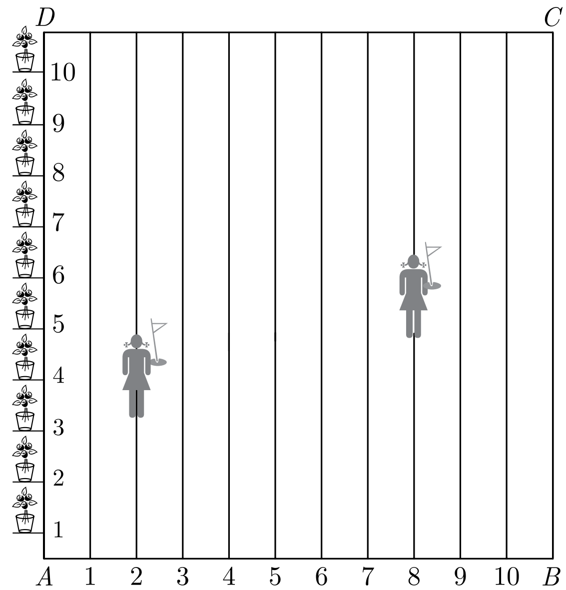
$$3x = 2y$$ Hence proved.

FOUR MARKS QUESTIONS

120. To conduct Sports Day activities, in your rectangular school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Figure. Niharika runs ¼th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th distance AD on the eighth line and posts a red flag.

(i) What is the distance between the two flags?

(ii) If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post the blue flag?



Ans : [Board 2020 Delhi Basic]

We assume A as origin (0, 0), AB as x-axis and AD as y-axis.

Niharika runs in the 2nd line with green flag and distance covered (parallel to AD),

$$= \frac{1}{4} \times 100 = 25 \text{ m}$$

Thus co-ordinates of green flag are (2, 25) and we label it as P i.e., $P(2, 25)$.

Similarly, Preet runs in the eighth line with red flag and distance covered (parallel to AD),

$$= \frac{1}{5} \times 100 = 20 \text{ m}$$

Co-ordinates of red flag are (8, 20) and we label it as Q i.e., $Q(8, 20)$

(i) Now, using distance formula, distance between green flag and red flag,

$$PQ = \sqrt{(8-2)^2 + (20-25)^2}$$

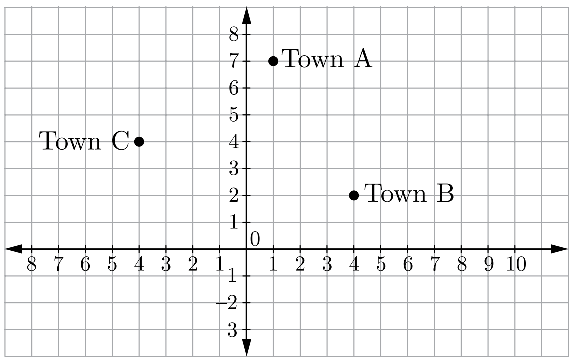
$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25}$$

$$= \sqrt{61} \text{ m}$$

(ii) Also, Rashmi has to post a blue flag the mid-point of PQ, therefore by using mid-point formula, we obtain $\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$ i.e. $\left(5, \frac{45}{2}\right)$

Hence, the blue flag is in the fifth line, at a distance of $\frac{45}{2}$ i.e., 22.5 m along the direction parallel to AD.

121. Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometown represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation answer the following questions:



(i) Who will travel more distance, Seema or Aditya, to reach to their hometown?

(ii) Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B. Find the coordinates of the point represented by the point D.

(iii) Find the area of the triangle formed by joining the points represented by A, B and C.

Ans : [Board 2020 SQP Standard]

From the given figure, the coordinates of points A, B and C are $(1, 7), (4, 2)$ and $(-4, 4)$ respectively.

(i) Distance travelled by seema

$$CA = \sqrt{(-4-1)^2 + (4-7)^2}$$
$$= \sqrt{(-5)^2 + (-3)^2}$$
$$= \sqrt{25+9} \qquad = \sqrt{34}$$

units

Thus distance travelled by seema is $\sqrt{34}$ units.
Similarly, distance travelled by Aditya

$$CB = \sqrt{(4+4)^2 + (4-2)^2}$$
$$= \sqrt{8^2 + 2^2} = \sqrt{64+4}$$
$$= \sqrt{68} \text{ units}$$

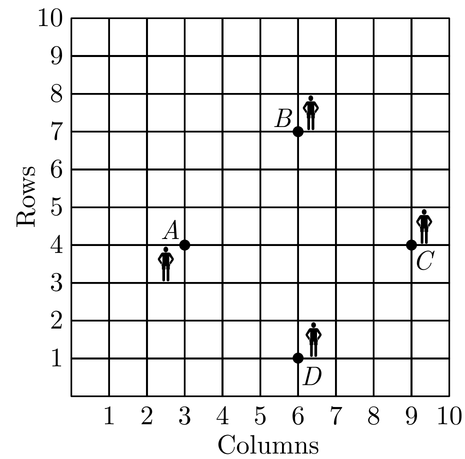
Distance travelled by Aditya is $\sqrt{68}$ units and Aditya travels more distance.

(ii) Since, D is mid-point of town A and town B

$$D = \left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$$

(iii) Removed from syllabus

122. In a classroom, 4 friends are seated at the points A, B, C, and D as shown in Figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, Don't you think $ABCD$ is a square? Chameli disagrees. Using distance formula, find which of them is correct.



Ans : [Board 2020 Delhi Basic]

Coordinates of points A, B, C, D are $A(3, 4)$, $B(6, 7)$, $C(9, 4)$ and $D(6, 1)$.

Distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Now
$$AB = \sqrt{(3-6)^2 + (4-7)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$BC = \sqrt{(6-9)^2 + (7-4)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$CD = \sqrt{(9-6)^2 + (4-1)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$DA = \sqrt{(6-3)^2 + (1-4)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

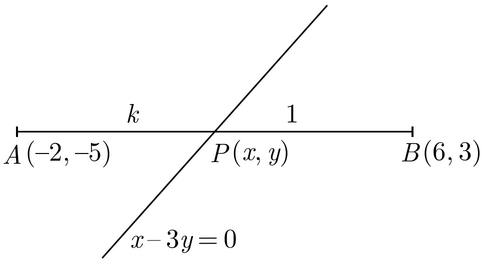
Now
$$AC = \sqrt{(3-9)^2 + (4-4)^2}$$
$$= \sqrt{36+0} = 6 \text{ units}$$
$$DB = \sqrt{(6-6)^2 + (1-7)^2}$$
$$= \sqrt{0+36} = 6 \text{ units}$$

Since, $AB = BC = CD = DA$ and $AC = DB$, $ABCD$ is a square and Champa is right.

123. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the coordinates of the point of intersection.

Ans : [Board 2019 OD]

Let $k:1$ be the ratio in which line $x - 3y = 0$ divides the line segment at $p(x, y)$.



Using section formula, we get

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{k \times 6 + 1 \times (-2)}{k+1}$$

$$x = \frac{6k-2}{k+1} \qquad ...(1)$$

and $$y = \frac{my_2 + ny_1}{m+n} = \frac{k \times 3 + 1 \times (-5)}{k+1}$$

$$y = \frac{3k-5}{k+1} \qquad ...(2)$$

The point $P(x, y)$ lies on the line, hence it satisfies the equation of the given line.

$$\frac{6k-2}{k+1} - 3\left(\frac{3k-5}{k+1}\right) = 0$$

$$6k - 2 - 3(3k-5) = 0$$

$$6k - 2 - 9k + 15 = 0$$

$$-3k + 13 = 0 \Rightarrow k = \frac{13}{3}$$

Hence, the required ratio is 13 : 3.

Now, substituting value of k in x and y, we get

$$x = \frac{6 \times \frac{13}{3} - 2}{\frac{13}{3} + 1} = \frac{78-6}{16} = \frac{72}{16} = \frac{9}{2}$$

$$y = \frac{3 \times \frac{13}{3} - 5}{\frac{13}{3} + 1} = \frac{8 \times 3}{16} = \frac{24}{16} = \frac{3}{2}$$

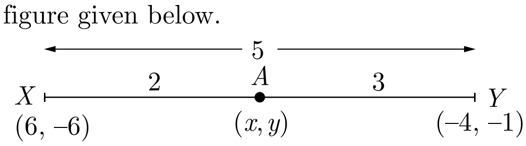
Hence, the co-ordinates of point of intersection

$$P(x, y) = \left(\frac{9}{2}, \frac{3}{2}\right)$$

124. Point A lies on the line segment XY joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line $3x + k(y+1) = 0$, find the value of k.

Ans : [Board 2019 OD]

As per given information in question we have drawn the figure given below.



We use section formula for point $A(x, y)$.

Here, $m_1 = 2$, $m_2 = 3$, $x_1 = 6$, $x_2 = -4$, $y_1 = -6$ and $y_2 = -1$

Now $$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2 \times (-4) + 3(6)}{2+3}$$

$$= \frac{-8+18}{5} = \frac{10}{5} = 2$$

and $$y = \frac{m_1y_2 + m_2y_2}{m_1 + m_2} = \frac{2 \times (-1) + 3(-6)}{2+3}$$

$$= \frac{-2-18}{5} = \frac{-20}{5} = -4$$

Hence, coordinates of point A is $(2, -4)$.

Since point A also lies on the line $3x + k(y+1) = 0$, its coordinates must satisfies this line.

Thus $$3(2) + k(-4+1) = 0$$

$$6 + (-3k) = 0$$

$$3k = 6 \Rightarrow k = 2$$

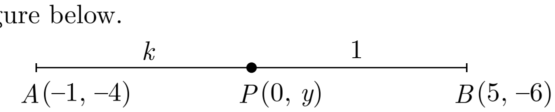
Hence, value of k is 2.

125. Find the ratio in which the y-axis divides the line segment joining the points $(-1, -4)$ and $(5, -6)$. Also find the coordinates of the point of intersection.

Ans : [Board 2019 OD]

Let points $P(0, y)$ divides the line joining the point $A(-1, -4)$ and $B(5, -6)$ in ratios $k:1$.

As per given information in question we have drawn figure below.



Section formula is given by

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \qquad ...(1)$$

and $$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \qquad ...(2)$$

Here, $m_1 = k$ and $m_2 = 1$,

$x_1 = -1$ and $x_2 = 5$

$y_1 = -4$ and $y_2 = -6$

Now $$0 = \frac{k \times 5 + 1 \times (-1)}{k+1}$$

$$5k - 1 = 0 \Rightarrow k = \frac{1}{5}$$

Substitute value of k in eq (2), we get

$$y = \frac{k(-6) + 1(-4)}{k+1}$$

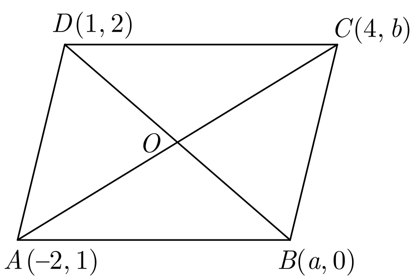
$$= \frac{\frac{1}{5}(-6) + 1(-4)}{\frac{1}{5} + 1} = \frac{-26}{6} = \frac{-13}{3}$$

Hence, value of k is $\frac{1}{5}$ and required point is $\left(0, -\frac{13}{3}\right)$

126. If $A(-2,1)$, $B(a,0)$, $C(4,b)$ and $D(1,2)$ are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides.

Ans : [Board 2018]

As per information given in question we have drawn the figure below.



Here $ABCD$ is a parallelogram and diagonals AC and BD bisect each other. Therefore mid point of BD is same as mid point of AC.

$$\left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{-2+4}{2}, \frac{b+1}{2}\right)$$

$$\frac{a+1}{2} = 1 \Rightarrow a = 1$$

and $$\frac{b+1}{2} = 1 \Rightarrow b = 1$$

Now
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1+2)^2 + (0-1)^2}$$
$$= \sqrt{9+1} = \sqrt{10} \text{ unit}$$
$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-1)^2 + (1-0)^2}$$
$$= \sqrt{9+1} = \sqrt{10} \text{ unit}$$

Since $ABCD$ is a parallelogram,

$$AB = CD = \sqrt{10} \text{ unit}$$
$$BC = AD = \sqrt{10} \text{ unit}$$

Therefore length of sides are $\sqrt{10}$ units each.

127. If $P(9a-2, -b)$ divides the line segment joining $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio 3:1. find the values of a and b.

Ans : [Board Term-2 SQP 2016]

Using section formula we have

$$9a - 2 = \frac{3(8a) + 1 + (3a+1)}{3+1} \quad ...(1)$$

$$-b = \frac{3(5) + 1(-3)}{3+1} \quad ...(2)$$

Form (2) $$-b = \frac{15-3}{4} = 3 \Rightarrow b = -3$$

From (1), $$9a - 2 = \frac{24a + 3a + 1}{4}$$

$$4(9a-2) = 27a + 1$$
$$36a - 8 = 27a + 1$$
$$9a = 9 \Rightarrow a = 1$$

128. Find the coordinates of the point which divide the line segment joining $A(2, -3)$ and $B(-4, -6)$ into three equal parts.

Ans : [Board Term-2 SQP 2016]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect the line joining $A(3, -2)$ and $B(-3, -4)$.

As per question, line diagram is shown below.

P divides AB in the ratio of 1:2 and Q divides AB in the ratio 2:1. 1

By section formula

$$x_1 = \frac{mx_2 + nx_1}{1+2} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$P(x_1, y_1) = \left(\frac{1(-4) + 2(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

$$= \left(\frac{-4+4}{3}, \frac{-6-(-6)}{3}\right) = (0, -4)$$

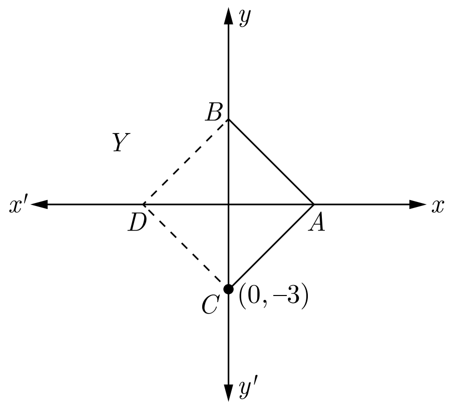
$$Q(x_2, y_2) = \left(\frac{2(-4) + 1(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

$$= \left(\frac{-8+2}{3}, -\frac{12+(-3)}{3}\right) = (-2, -5)$$

129. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of point C are $(0,3)$. The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that $BACD$ is a rhombus.

Ans : [Board Term-2 Foreign 2015]

As per question, diagram of rhombus is shown below.



Co-ordinates of point B are $(0,3)$.

Thus $BC = 6$ unit

Let the co-ordinates of point A be $(x,0)$

Now $AB = \sqrt{x^2+9}$

Since $AB = BC$, thus we have

$$x^2+9 = 36$$

$$x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

Co-ordinates of point A is $(3\sqrt{3},0)$.

Since $ABCD$ is a rhombus,

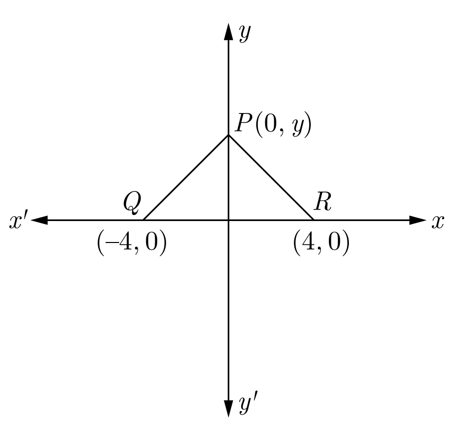
$$AB = AC = CD = DB$$

Thus co-ordinate of point D is $(-3\sqrt{3},0)$.

130. The base QR of an equilateral triangle PQR lies on x-axis. The co-ordinates of point Q are $(-4,0)$ and the origin is the mid-point of the base. find the co-ordinates of the point P and R.

Ans : [Board Term-2 Delhi 2017, Foreign 2015]

As per question, line diagram is shown below.



Co-ordinates of point R is $(4,0)$.

Thus $QR = 8$ units

Let the co-ordinates of point P be $(0,y)$

Since $PQ = QR$

$$(-4-0)^2+(0-y)^2 = 64$$

$$16+y^2 = 64$$

$$y = \pm 4\sqrt{3}$$

Coordinates of P are $(0,4\sqrt{3})$ or $(0,-4\sqrt{3})$

131. The vertices of quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)$ and $D(1,-4)$. Prove that $ABCD$ is a rhombus.

Ans : [Board Term-2 Delhi 2015]

The vertices of the quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)\,D(1,-4)$.

Now $AB = \sqrt{(8-5)^2+(3+1)^2}$

$= \sqrt{3^2+4^2} = 5$ units

$BC = \sqrt{(8-4)^2+(3-0)^2}$

$= \sqrt{4^2+3^2} = 5$ units

$CD = \sqrt{(4-1)^2+(0+4)^2}$

$= \sqrt{(3)^2+(4)^2} = 5$ units

$AD = \sqrt{(5-1)^2+(-1+4)^2}$

$= \sqrt{(4)^2+(3)^2} = 5$ units

Diagonal, $AC = \sqrt{(5-4)^2+(-1-0)^2}$

$= \sqrt{1^2+1^2} = \sqrt{2}$ units

Diagonal $BD = \sqrt{(8-1)^2+(3+4)^2}$

$= \sqrt{(7)^2+(7)^2} = 7\sqrt{2}$ units

As the length of all the sides are equal but the length of the diagonals are not equal. Thus $ABCD$ is not square but a rhombus.

132. The co-ordinates of vertices of ΔABC are $A(0,0)$, $B(0,2)$ and $C(2,0)$. Prove that ΔABC is an isosceles triangle. Also find its area.

Ans : [Board Term-2 Delhi 2014]

Using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ we have

$$AB = \sqrt{(0-0)^2 + (0-2)^2} = \sqrt{4} = 2$$
$$AC = \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4} = 2$$
$$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Clearly, $AB = AC \neq BC$

Thus ΔABC is an isosceles triangle.

Now, $AB^2 + AC^2 = 2^2 + 2^2 = 4 + 4 = 8$

also, $BC^2 = (2\sqrt{2})^2 = 8$

$$AB^2 + AC^2 = BC^2$$

Thus ΔABC is an isosceles right angled triangle.

Now, area of ΔABC

$$\Delta_{ABC} = \frac{1}{2}\text{base} \times \text{height}$$
$$= \frac{1}{2} \times 2 \times 2$$
$$= 2 \text{ sq. units.}$$
$$= \frac{1}{2}[3 \times (-1) + 7 \times 2 + 5 \times (-1)]$$
$$= \frac{1}{2}[-3 + 14 - 5]$$
$$= 3 \text{ units}$$

Area $\square_{ABCD} = \frac{5}{2} + 3 = \frac{11}{2}$ sq. units.

133. Find the ratio is which the line segment joining the points $A(3, -3)$ and $B(-2, 7)$ is divided by x-axis. Also find the co-ordinates of the point of division.

Ans : [Board Term-2 OD 2014]

We have $A(3, -3)$ and $B(-2, 7)$.
At any point on x-axis y-coordinate is always zero.
So, let the point be $(x, 0)$ that divides line segment AB in ratio $k:1$.

Now $$(x,\ 0) = \left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$$

$$\frac{7k-3}{k+1} = 0$$
$$7k - 3 = 0 \Rightarrow k = \frac{3}{7}$$

The line is divided in the ratio of 3 : 7.

Now $$\frac{-2k+3}{k+1} = x$$

$$\frac{-2 \times \frac{3}{7} + 3}{\frac{3}{7} + 1} = x$$
$$\frac{-6+21}{3+7} = x$$
$$\frac{15}{10} = x \Rightarrow x = \frac{3}{2}$$

The coordinates of the point is $\left(\frac{3}{2}, 0\right)$.

134. Determine the ratio in which the straight line $x - y - 2 = 0$ divides the line segment joining $(3, -1)$ and $(8, 9)$.

Ans : [Board Term-2, 2012]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio $k:1$.

Now $$x_1 = \frac{8k+3}{k+1}$$
$$y_1 = \frac{9k-1}{k+1}$$

Since point $P(x_1, y_1)$ lies on line $x - y - 2 = 0$, so co-ordinates of P must satisfy the equation of line.

Thus $$\frac{8k+3}{k+1} - \frac{9k-1}{k+1} - 2 = 0$$
$$8k + 3 - 9k + 1 - 2k - 2 = 0$$
$$-3k + 2 = 0 \Rightarrow k = \frac{2}{3}$$

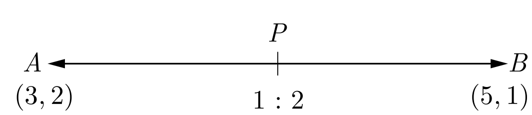
So, line $x - y - 2 = 0$ divides AB in the ratio 2:3

135. The line segment joining the points $A(3, 2)$ and $B(5, 1)$ is divided at the point P in the ratio 1:2 and P lies on the line $3x - 18y + k = 0$. Find the value of k.

Ans : [Board Term-2 Delhi 2012]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio $1:2$.

2



$$x_1 = \frac{mx_2 + nx_1}{m+n} = \frac{1 \times 5 + 2 \times 3}{1+2} = \frac{11}{3}$$
$$y_2 = \frac{my_2 + ny_1}{m+n} = \frac{1 \times 2 + 2 \times 2}{1+2} = \frac{5}{3}$$

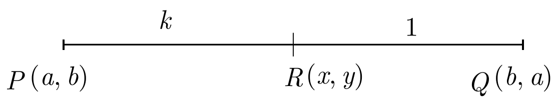
Since point $P(x_1, y_1)$ lies on line ,$3x - 18y + k = 0$, so co-ordinates of P must satisfy the equation of line.

$$3 \times \frac{11}{3} - 18 \times \frac{5}{3} + k = 0$$
$$k = 19$$

136. If $R(x, y)$ is a point on the line segment joining the points $P(a, b)$ and $Q(b, a)$, then prove that $x + y = a + b$.

Ans : [Board Term-2, 2012 Set (28)]

As per question line is shown below.



Let point $R(x, y)$ divides the line joining P and Q in the ratio $k:1$, then we have

$$x = \frac{kb + a}{k + 1}$$

and $$y = \frac{ka + b}{k + 1}$$

Adding, $$x + y = \frac{kb + a + ka + b}{k + 1}$$

$$= \frac{k(a + b) + (a + b)}{k + 1}$$

$$= \frac{(k + 1)(a + b)}{k + 1} = a + b$$

$$x + y = a + b$$ Hence Proved

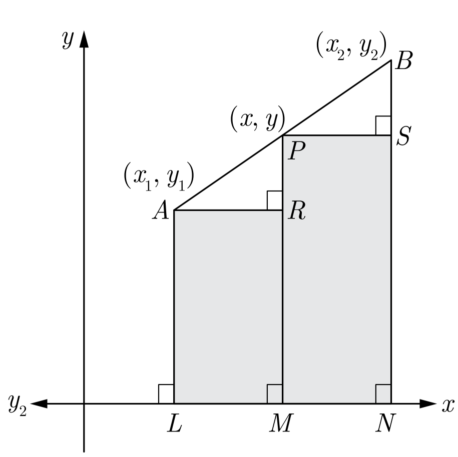
137. (i) Derive section formula.

(ii) In what ratio does $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$

Ans : [Board Term-2 Delhi 2014]

(i) Section Formula : Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points. Let $P(x, y)$ be a point on line, joining A and B, such that P divides it in the ratio $m_1 : m_2$.

Now $$(x, y) = \left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$$



Proof : Let AB be a line segment joining the points. $A(x_1, y_1)$, $B(x_2, y_2)$.

Let P divides AB in the ratio $m_1 : m_2$. Let P have co-ordinates (x, y).

Draw $AL, PM, PN, \perp$ to x-axis

It is clear form figure, that

$$AR = LM = OM - OL = x - x_1$$

$$PR = PM - RM = y - y_1.$$

also, $$PS = ON - OM = x_2 - x$$

$$BS = BN - SN = y_2 - y$$

Now $$\Delta APR \sim \Delta PBS$$ [AAA]

Thus $$\frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$$

and $$\frac{AR}{PS} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2}$$

$$m_2 x - m_2 x_1 = m_1 x_2 - m_1 x$$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

Now $$\frac{PR}{BS} = \frac{AP}{PB}$$

$$\frac{y - y_2}{y_2 - y} = \frac{m_1}{m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Thus co-ordinates of P are $\left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$

(ii) Assume that $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$ in ratio $k:1$

Using section formula for x co-ordinate we have

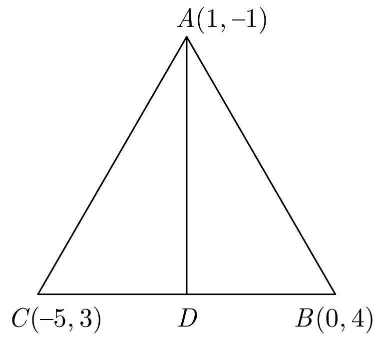
$$-4 = \frac{k(3) - 6}{k + 1}$$

$$-4k - 4 = 3k - 6 \Rightarrow k = \tfrac{2}{7}$$

138. $(1, -1)$, $(0, 4)$ and $(-5, 3)$ are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex $(1, -1)$ the mid-point of the opposite side.

Ans : [Board Term-2, 2015]

Let the vertices of ΔABC be $A(1, -1)$, $B(0, 4)$ and $C(-5, 3)$. Let $D(x, y)$ be mid point of BC. Now the triangle is shown below.



Using distance formula, we get

$$AB = \sqrt{(1-0)^2+(-1-4)^2} = \sqrt{1+5^2} = \sqrt{26}$$

$$BC = \sqrt{(-5-0)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(-5-1)^2+(3+1)^2} = \sqrt{36+16} = 2\sqrt{13}$$

Since $AB = BC \neq AC$, triangle ΔABC is isosceles.

Now, using mid-section formula, the co-ordinates of mid-point of BC are

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$

$$y = \frac{3+4}{2} = \frac{7}{2}$$

$$D(x,y) = \left(-\frac{5}{2}, \frac{7}{2}\right)$$

Length of median AD,

$$AD = \sqrt{\left(\frac{-5}{2}-1\right)^2+\left(\frac{7}{2}+1\right)^2}$$

$$= \sqrt{\left(\frac{-7}{2}\right)^2+\left(\frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2} \text{ square unit}$$

Thus length of median AD is $\frac{\sqrt{130}}{2}$ units.

139. Point $(-1, y)$ and $B(5,7)$ lie on a circle with centre $O(2, -3y)$. Find the values of y. Hence find the radius of the circle.

Ans : [Board Term-2 Delhi 2014]

Since, $A(-1, y)$ and $B(5,7)$ lie on a circle with centre $O(2, -3y)$, OA and OB are the radius of circle and are equal. Thus

$$OA = OB$$

$$\sqrt{(-1-2)^2+(y+3y)^2} = \sqrt{(5-2)^2+(7+3y)^2}$$

$$9+16y^2 = 9y^2+42y+58$$

$$y^2-6y-7 = 0$$

$$(y+1)(y-7) = 0$$

$$y = -1, 7$$

When $y = -1$, centre is $O(2, -3y) = (2,3)$ and radius

$$OB = \left|\sqrt{(5-2)^2+(7-3)^2}\right|$$

$$= \sqrt{9+16} = 5 \text{ unit}$$

When $y = 7$, centre is $O(2, -3y) = (2, -21)$ and radius

$$OB = \left|\sqrt{(2-5)^2+(-21-7)^2}\right|$$

$$= \left|\sqrt{9+784}\right| = \sqrt{793} \text{ unit}$$